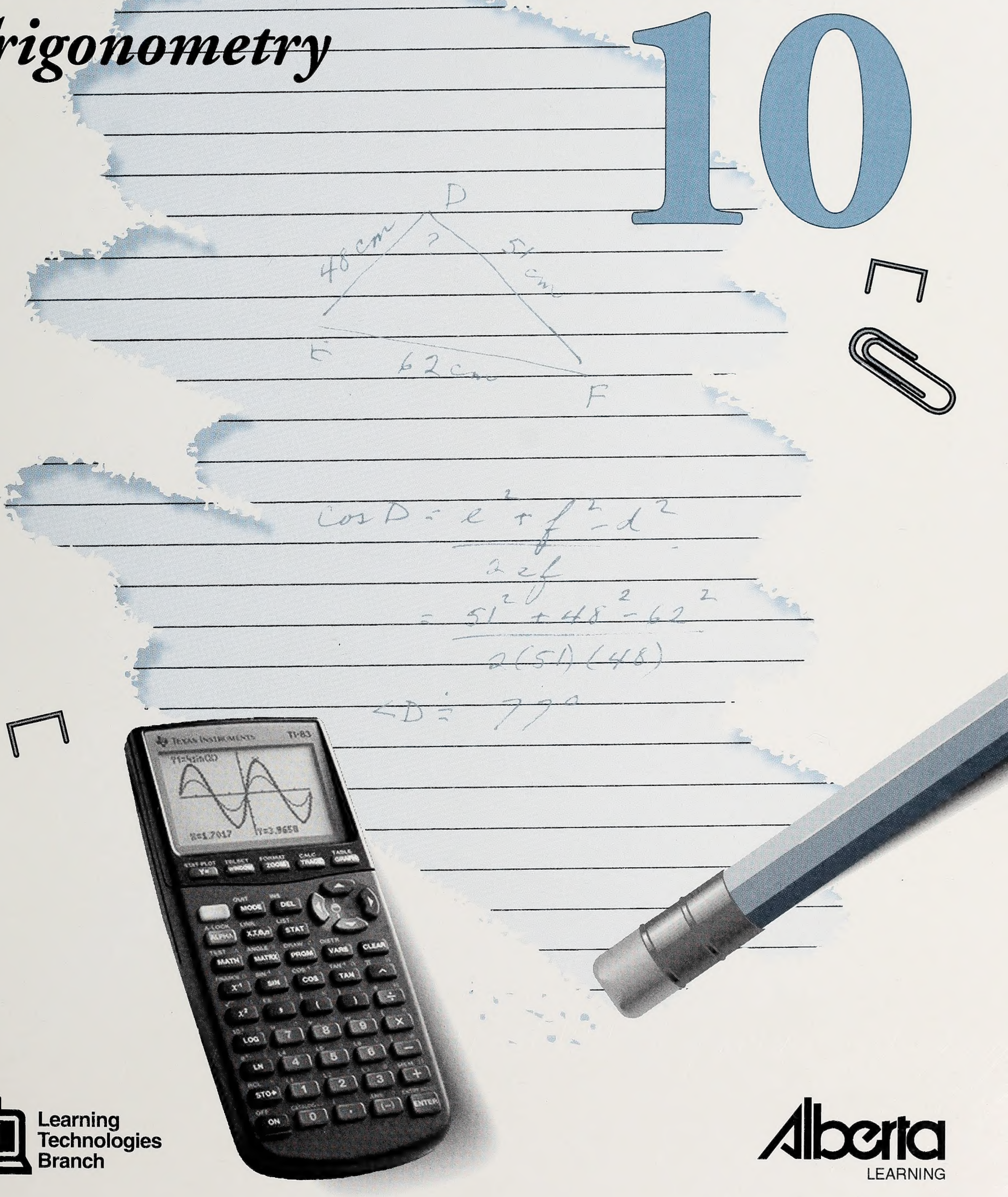
University of Alberta Library
0 1620 3410379 4

Module 5

MATHEMATICS 10

Trigonometry



Learning
Technologies
Branch

Alberta
LEARNING

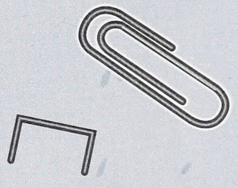
Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

https://archive.org/details/puremathematics105albe

Pure MATHEMATICS 10

Module 5

Trigonometry



Learning
Technologies
Branch

Alberta
LEARNING

Pure Mathematics 10
Module 5: Trigonometry
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2148-3

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

Copyright © 2002, the Crown in Right of Alberta, as represented by the Minister of Learning, Alberta Learning, 11160 Jasper Avenue, Edmonton, Alberta T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Learning.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Learning so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Welcome

Welcome to Module 5. We hope you'll enjoy your study of Trigonometry.

Pure Mathematics 10

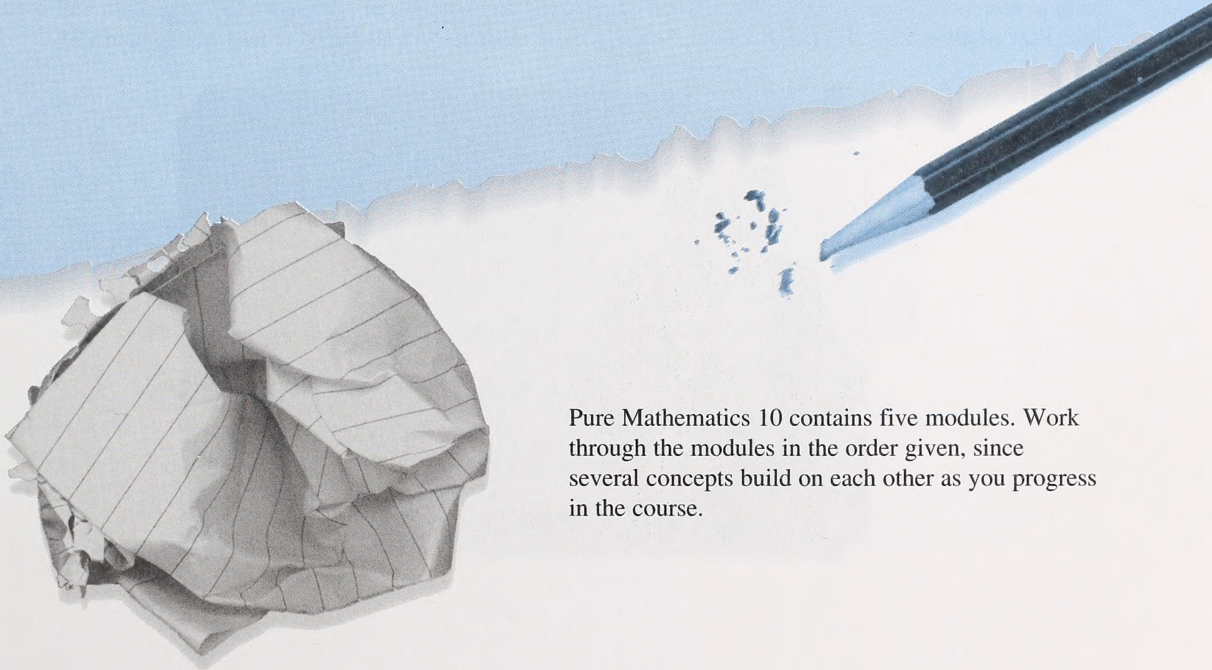
Module 1
Statistics and Data Tables

Module 2
Patterns and Relations

Module 3
Algebra

Module 4
Number Connections

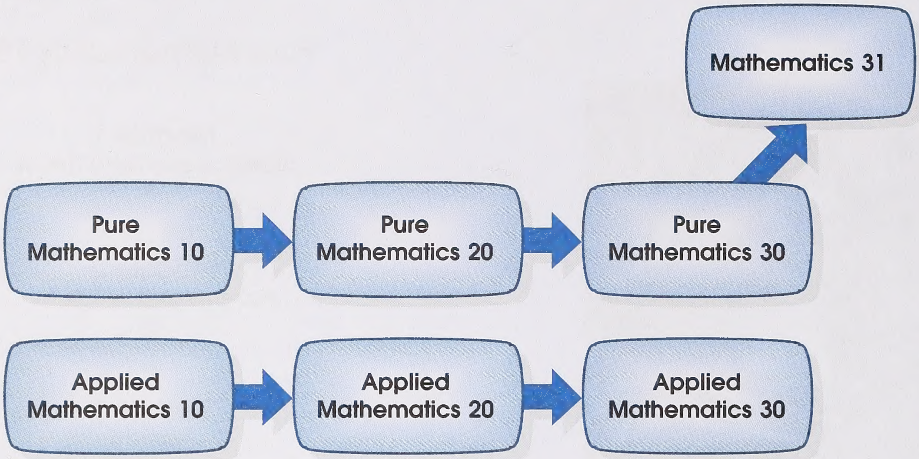
Module 5
Trigonometry



Pure Mathematics 10 contains five modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

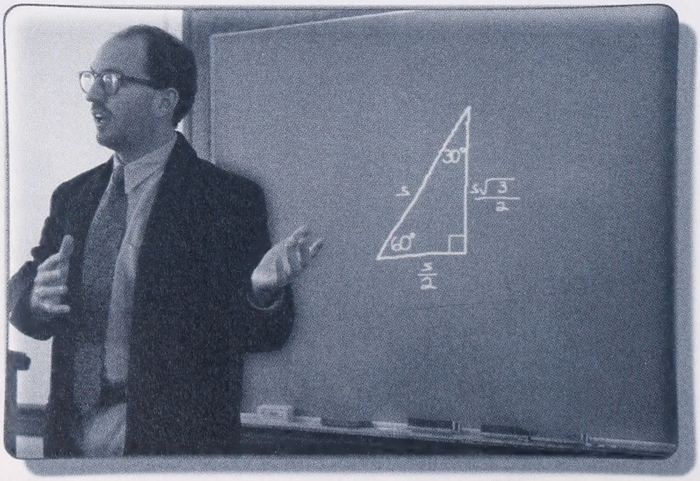
Pure Mathematics 10 is the first course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.

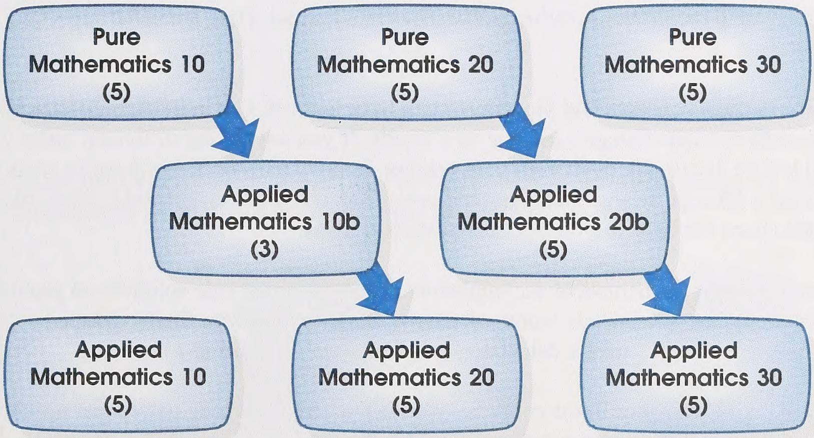


Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you will take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an "aha!") or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 10 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Centre:

- the *MATHPOWER™ 10* textbook and data disks, Western Edition, published by McGraw-Hill Ryerson (1998)

- a graphing calculator
  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* Algebra FX-2.0, CFX-9850 GA-Plus, CFX-9850 G, CFX-9800 G, or FX-9700 series; *Sharp* EL-9600C, EL-9600, EL-9200, or EL-9300; or *Hewlett-Packard* HP 39g (only approved for 2001–2003 school years). **Note:** Some calculators are no longer commercially manufactured.

- spreadsheet program
  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh or for Windows, *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable. The screen shots in this module have been reprinted by permission from Apple Computer, Inc.

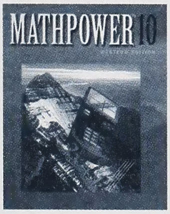
You will need access to a computer (mandatory) and a videocassette player (optional).

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

View a videocassette.

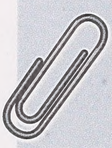
Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Use the Computer Data Bank.

Remember: *Any Internet website address given in this module is subject to change.*

CONTENTS



Module Overview

Evaluation 2
Strategies for Completing a Module 3

Section 1

Measurement

Activity 1: Reviewing Trigonometric Ratios 6
Activity 2: Problems Involving Two Right Triangles 13
Activity 3: Angles in Standard Position 17
Follow-up Activities 22
Extra Help 22
Enrichment 24
Conclusion 25
Assignment 25

Section 2

The Laws of Sines and Cosines

Activity 1: The Law of Sines 28
Activity 2: The Law of Cosines 32
Follow-up Activities 38
Extra Help 38
Enrichment 42
Conclusion 44
Assignment 44

Module Summary

Final Module Assignment 45
Course Survey

Appendix

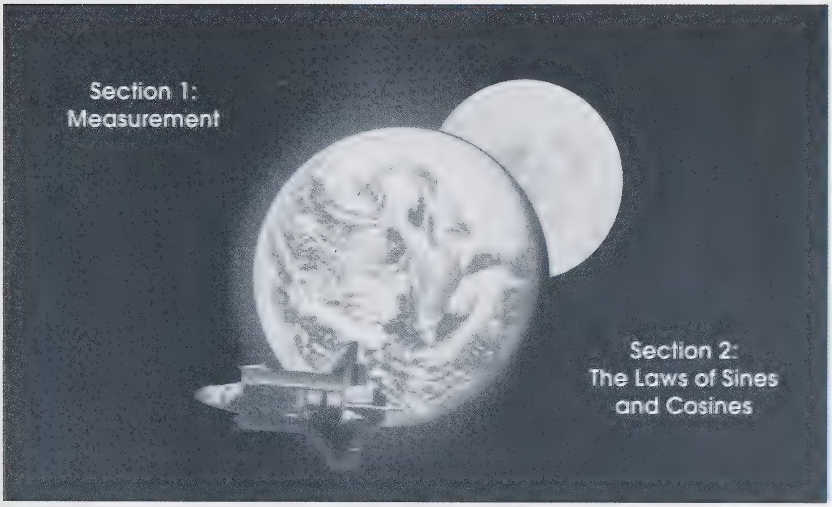
Glossary 48
Suggested Answers 48

Module Overview

The origins of mathematics, simply put, lie in questions such as "How many?", "How much?", "How long?", "How big?", and "How far?" For instance, these questions reflect the practical concerns of the ancient Egyptians in recordkeeping, in their calendar, in surveying the valley of the Nile, in constructing the pyramids, and so on. Today, these same questions come together in the quest to explore the universe.

In Section 1 you will investigate the basic techniques of trigonometry to help answer the questions of "How big?" and "How far?" In Section 2 you will further explore techniques of trigonometry by investigating the laws of sines and cosines.

Module 5: Trigonometry



Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Trigonometry, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module, you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based in part upon the assignments you submit for evaluation. The mark distribution is as follows:

Assignment Booklet 5A	
Section 1 Assignment	**30 marks**
Assignment Booklet 5B	
Section 2 Assignment	**30 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

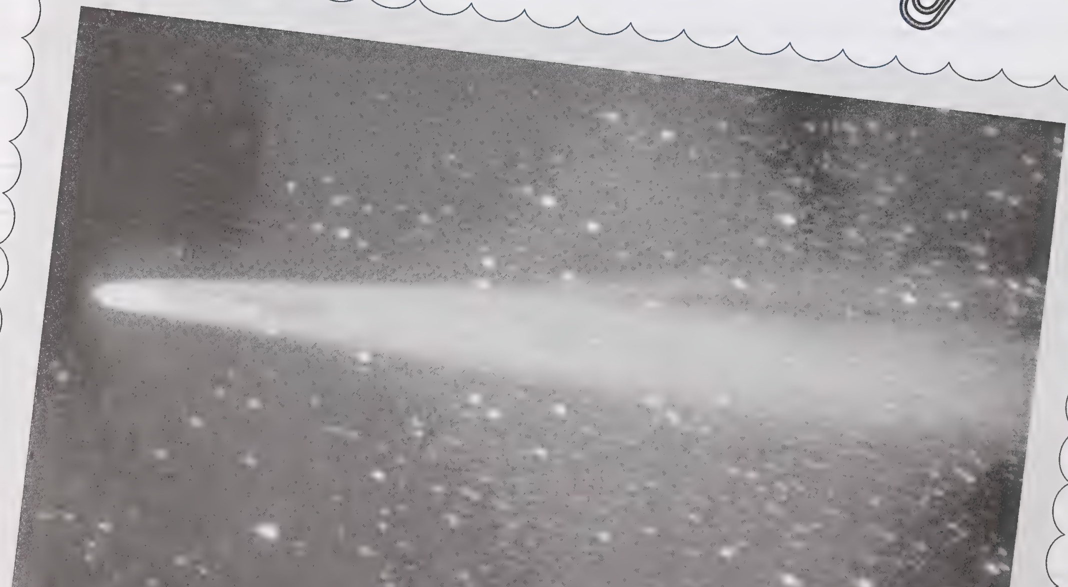
Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 10.

Good luck!

Section 1

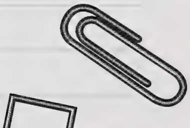


Measurement

Have you ever looked into the night sky and wondered about the distances to the stars or thought about how far certain planets, asteroids, or comets are from Earth? During the day, have you ever gazed at a distant mountain, at a shoreline across a river or lake, or at a cloud formation and wondered how far they were from you?

Trigonometry provides the mathematical tools for measuring the sizes or distances of inaccessible objects. From the Greek words for “triangle measurement,” modern trigonometry encompasses far more than just finding missing sides or angles of triangles. Trigonometry is used in surveying and construction and in the sciences—particularly physics and astronomy. Trigonometry is critical to furthering your understanding of higher mathematics.

In this section, you will review right-triangle trigonometry, solve problems involving the combinations of right triangles, and extend your work to include angles larger than 90°.

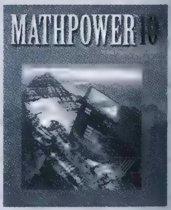


Activity 1: Reviewing Trigonometric Ratios

The dump truck in the photograph has raised its box to empty clay fill for a highway project. The higher the front of the box is raised, the greater the angle between the box and the horizontal becomes. To determine the angle between the box and the horizontal, you can use right-triangle trigonometry if you know the length of the box and the length of the exposed portion of the hydraulic lift.

In this activity, you will review the trigonometric ratios obtained from the sides of a right triangle and employ these ratios in solving related problems.

Mathematical Process
- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization



Turn to "INVESTIGATING MATH" on page 328 of *MATHPOWER™ 10* and review the definitions of the trigonometric ratios in Investigation 1, "Sine, Cosine, and Tangent."

1. Answer the following questions on pages 328 and 329 of the textbook:

   a. questions 1 and 2 of Investigation 1, "Sine, Cosine, and Tangent"

   b. questions 1 to 3 of Investigation 2, "Special Right Triangles"

   c. questions 1 and 2 of Investigation 3, "Problems Involving Two Special Right Triangles"

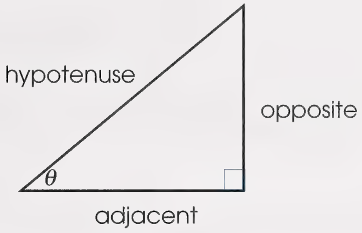
Compare your responses with those in the Appendix, Section 1: Activity 1.

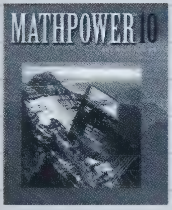
The trigonometric ratios (**sine**, **cosine**, and **tangent**) can be recalled using the memory aid **SOH•CAH•TOA**, which sounds like "soak a toe, eh!"—something you may have to do if you trip over a right triangle!

$$\textbf{SOH}: \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\textbf{CAH}: \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\textbf{TOA}: \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$





Turn to page 330 of *MATHPOWER™ 10* and read "Reviewing Trigonometric Ratios."

2. Answer the following questions on page 330 of the textbook:

a. "Explore: Draw a Diagram"
b. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 1.

There are two main types of problems involving trigonometric ratios and the right triangle:

- You are given the measure of one of the acute angles and the length of a side, and you are expected to find the missing measures (sides and/or angles).
- You are given the lengths of two sides of the right triangle, and you are expected to find the missing measures.

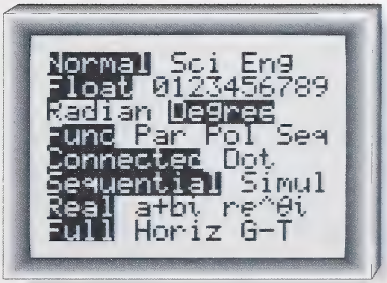
solving a right triangle: determining the lengths of its sides and the measures of its angles

When **solving a right triangle**—that is, finding the missing sides and angles—you will work with sine, cosine, and tangent on your graphing calculator.

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
☒ Technology
■ Visualization

Because angles will be given in degrees, set your calculator to degree mode. Press MODE; highlight "Degree" using the arrow keys to move the cursor; and press ENTER.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

You are now ready to use the trigonometric function keys.

Example

Using a graphing calculator, find the following:

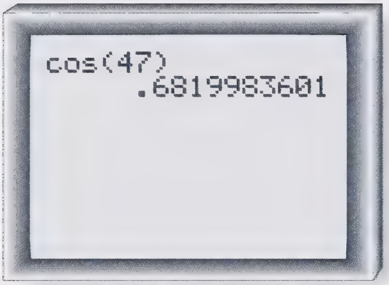
a. cos 47°
b. the acute angle, A, such that tan $A = 0.3456$

Solution

a. To evaluate cos 47°, press the following

$\boxed{\text{cos}}\ \boxed{4}\ \boxed{7}\ \boxed{)}\ \boxed{\text{ENTER}}$

The calculator will display the following:

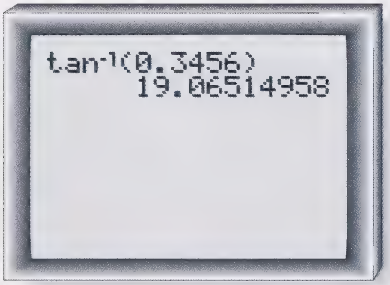


```
cos(47)
         .6819983601
```

b. To work from the ratio to the angle, press the following:

$\boxed{\text{2nd}}\ [\tan^{-1}]\ \boxed{0}\ \boxed{\bullet}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{)}\ \boxed{\text{ENTER}}$

The calculator will display the following:



```
tan⁻¹(0.3456)
         19.06514958
```

This means that if $\angle A = 19.065\,149\,58$, then $\tan A = 0.3456$.

You can check this by pressing ☐ ☐ [AN] ☐ EN ER.

Tables were used prior to calculators and computers. A Greek mathematician by the name of Hipparchus, who is considered to be the father of trigonometry, developed the first trigonometric tables using chords of circles in 140 B.C. In the second century, Ptolemy, of Alexandria, developed tables accurate to 5 and 6 figures working with chords as well.

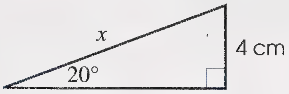
If you have access to the Internet, you can learn more about these mathematicians at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Hipparchus.html
http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Ptolemy.html

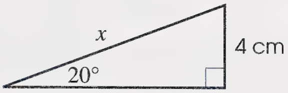
You will now practise using your calculator to solve questions involving the trigonometric ratios.

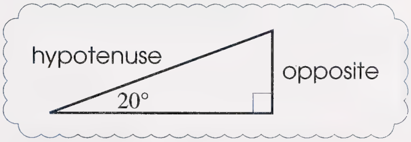
Example

In the triangle given, calculate x to the nearest tenth of a unit.



Solution





The 4-cm side, which lies directly across from the given angle, is the side **opposite**. The length of the **hypotenuse** is represented by x.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

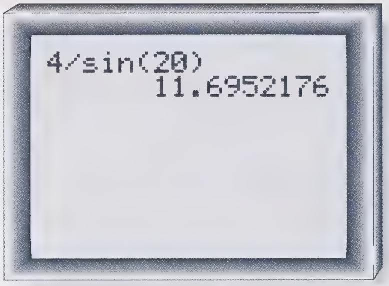
Substitute the given values into the formula.

$$\sin 20° = \frac{4}{x}$$

$$x \sin 20° = 4$$

$$x = \frac{4}{\sin 20°}$$

4 ÷ sin 2 0) ENTER

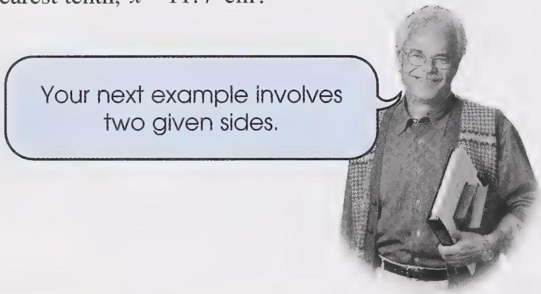


```
4/sin(20)
         11.6952176
```

Mathematical Process

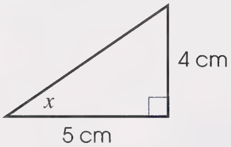
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Rounded to the nearest tenth, $x = 11.7$ cm.

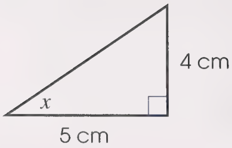

Your next example involves two given sides.

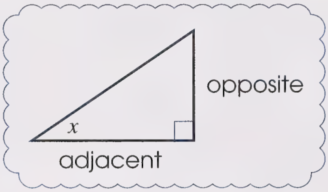
Example

In the triangle given, calculate $\angle x$ to the nearest degree.



Solution





The 4-cm side, which is directly across from the unknown angle, is the side **opposite**. The 5-cm side, which is next to the unknown angle and is the other leg forming the right angle, is the side **adjacent**.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the given values into the formula.

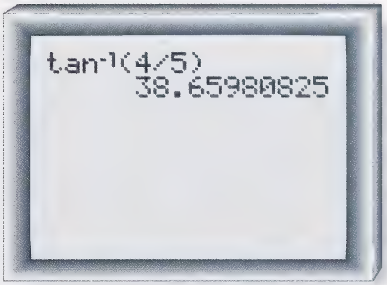
$$\tan x = \frac{4}{5}$$

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ☒ Technology
- ■ Visualization

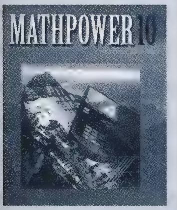
2nd [$\tan^{-1}$] 4 ÷ 5)

ENTER

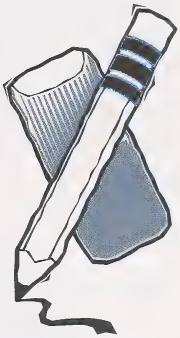


```
tan⁻¹(4/5)
         38.65980825
```

To the nearest degree, $\angle x = 39°$.



3. Turn to page 329 of *MATHPOWER™ 10* and answer the following:

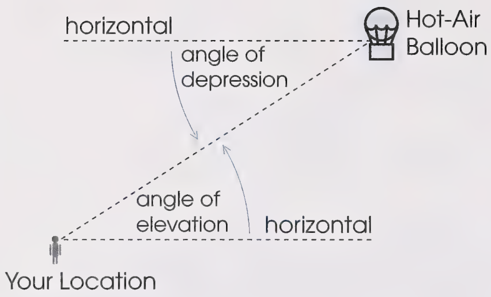


a. questions 1 and 2 of Investigation 4, "Trigonometric Ratios of Any Acute Angle"

b. questions 1, 3, and 6 of Investigation 5, "Finding Side Lengths in Any Right Triangle"

c. questions 1 to 3 of Investigation 6, "Finding Angles in Any Right Triangle"

Compare your responses with those in the Appendix, Section 1: Activity 1.



In many problems involving the right triangle, you will be working with **angles of elevation** and **angles of depression**. Both angles of elevation and depression are measured from the horizontal and depend on the location of the observer.

If you were looking up at a hot-air balloon, the angle through which you would have to raise your eyes from the horizontal to see the balloon is called the angle of elevation. A passenger in the balloon would have to look down from the horizontal to see you. That angle is called the angle of depression. The angle of elevation and angle of depression are always equal in measure!

Turn to pages 330 and 331 of *MATHPOWER*™ *10* and work through Examples 1 to 3.

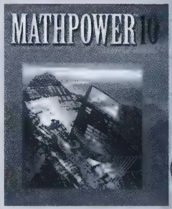
4. Answer the following questions on page 332 of the textbook:

a. questions 2, 4, 6, and 8 of "Practice"
b. questions 9, 10, 11, 15, and 17 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

angle of elevation: the angle through which an observer looks upward from the horizontal to see an object above

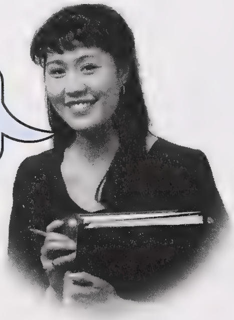
angle of depression: the angle through which an observer looks downward from the horizontal to see an object below



Now Try This

Problems in trigonometry involve spatial reasoning and the ability to visualize in three dimensions. Angles of elevation and depression, for example, depend on the observer's point of view.

Here is a puzzle that challenges your problem-solving skills.



5. Turn to page 337 of *MATHPOWER*™ *10* and answer questions a. to d. of "LOGIC POWER."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process
☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☒ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Looking Back

In this activity, you reviewed the sine, cosine, and tangent ratios as defined in terms of the lengths of the sides of a right triangle. You then applied these ratios in problem-solving situations.

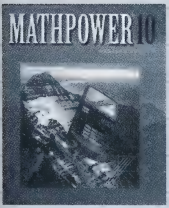
Trigonometry, like other areas of mathematics, has its own specialized vocabulary. In your journal, begin a list of the terms, along with their definitions, that were new to you in this activity.

Mathematical Process
- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 2: Problems Involving Two Right Triangles

Trigonometry, as you have studied, can be used to find the distances or heights of inaccessible objects, such as the cloud bank in the photograph. In the examples in this activity, you will solve problems like these using various combinations of two or more right triangles. Carefully drawn and labelled diagrams will prove indispensable.

Mathematical Process
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



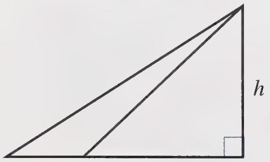
Turn to page 333 of *MATHPOWER™ 10* and read "Problems Involving Two Right Triangles," including "Explore: Use a Diagram."

1. Answer questions 1 to 3 of "Inquire" on page 333 of the textbook.

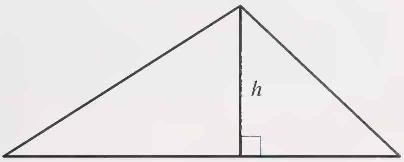
Compare your responses with those in the Appendix, Section 1: Activity 2.

The pairs of triangles that will be used to solve problems in this activity will often lie on the same side of a common altitude, h, or on opposite sides of a common altitude, h.

Same Side



Opposite Sides



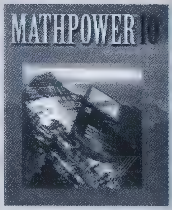
If the triangles lie on opposite sides, more often than not that common altitude is **not** a halfway point. Ensure that you read the problem carefully; then sketch a diagram as close to scale as possible so you can refer to your diagram to set up your solution and to check if your answer is reasonable.

Turn to pages 334 and 335 of *MATHPOWER*™ *10* and work through Examples 1 and 2.

2. Answer questions 2, 4, 6, and 8 of "Practice" on pages 335 and 336.

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

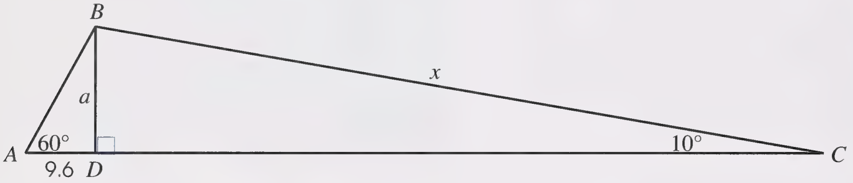


Compare your responses with those in the Appendix, Section 1: Activity 2.

When solving problems involving two right triangles, it is best to leave the final calculation to the end in order to avoid introducing rounding errors.

Example

In the diagram given, find x to the nearest tenth.

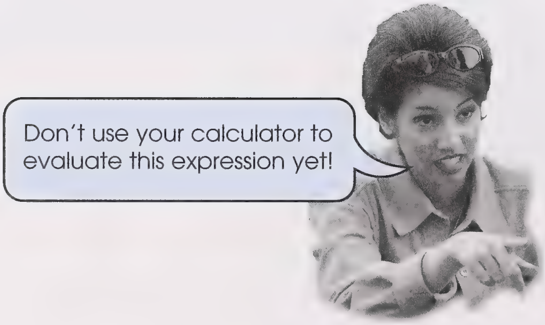


Solution

Use ΔABD to find an expression for a.

$$\tan 60° = \frac{a}{9.6}$$

$$a = 9.6 \tan 60°$$


Don't use your calculator to evaluate this expression yet!

Next, find an expression for x in terms of a.

$$\sin 10° = \frac{a}{x}$$

$$x = \frac{a}{\sin 10°}$$

Now, substitute for a.

$$x = \frac{9.6 \tan 60°}{\sin 10°}$$

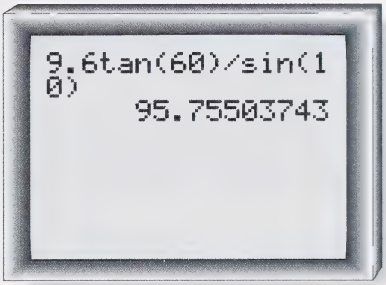
Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ☒ Technology
- ■ Visualization

Finally, evaluate using your calculator. Round your answer to one decimal place.

9 • 6 tan 6 0)

÷ sin 1 0) ENTER



```
9.6tan(60)/sin(1
0)
       95.75503743
```

Therefore, $x \doteq 95.8$.

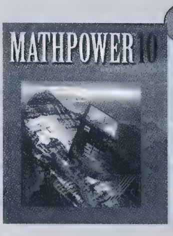
If, however, you had calculated a and had rounded it to one decimal place prior to substituting it into the expression for x, the value for x would have been as follows:

$$\begin{aligned} a &= 9.6 \tan 60° \\ &\doteq 16.6 \end{aligned}$$

$$\begin{aligned} x &= \frac{a}{\sin 10°} \\ &\doteq \frac{16.6}{\sin 10°} \\ &\doteq 95.6 \end{aligned}$$

Once again, to avoid rounding errors, round at the end, not at intermediate steps in the process.

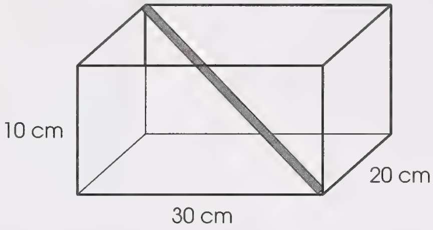
3. Turn to pages 336 and 337 of *MATHPOWER™ 10* and answer questions 9, 11, 12, 14, and 18 of "Applications and Problem Solving."



Compare your responses with those in the Appendix, Section 1: Activity 2.

Now Try This

4. What is the longest rod, to the nearest tenth of a centimetre, that can be placed inside an open box that is 20 cm by 30 cm by 10 cm? What angle, to the nearest tenth of a degree, would that rod make with the bottom of the box?



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Compare your responses with those in the Appendix, Section 1: Activity 2.

Looking Back

In this activity, you solved problems involving two right triangles. The right triangles could lie in the same plane or in three dimensions.

In your journal, write out a problem that involves two right triangles in three dimensions. Have a friend who is also taking Pure Mathematics 10 solve your problem.

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

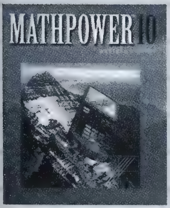
Activity 3: Angles in Standard Position

The triangular gable end of the barn in the photograph consists of two acute angles at the eaves and an obtuse angle at the ridge. Is it possible to obtain a trigonometric relationship among the sides and angles of this triangle? Can you find the trigonometric ratios of an obtuse angle?

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

In this activity, you will extend the concepts of sine and cosine to any angle between 0° and 180°.

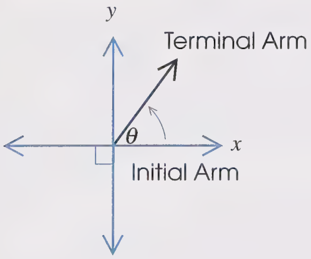


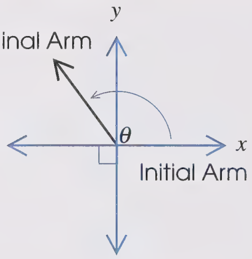
Turn to page 338 of *MATHPOWER™ 10* and read “Angles in Standard Position.”

1. Answer the following questions on pages 338 and 339 of the textbook:
   - **a.** “Explore: Look for Patterns”
   - **b.** questions 1 to 7 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 3.

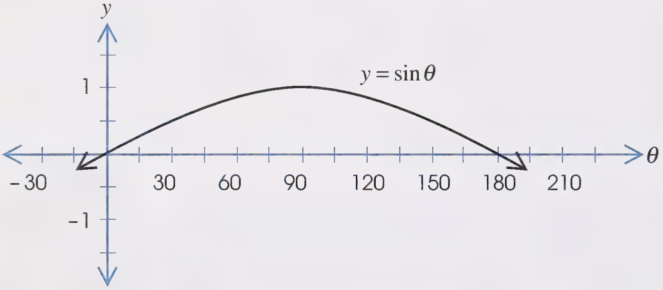
Your calculator determines the same values for sine and cosine as you would obtain from angles drawn in **standard position**.

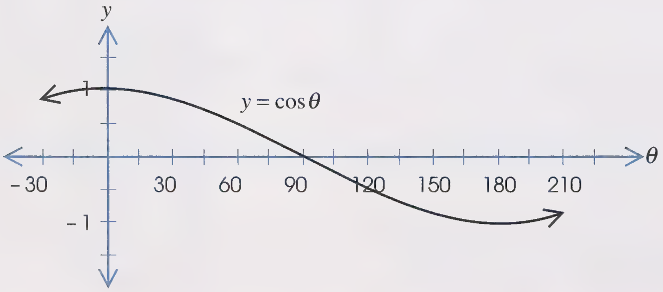




Remember that the **initial arm** of an angle in standard position lies along the positive x-axis, the vertex of the angle is located at the origin, and the **terminal arm** is free to rotate. As the terminal arm rotates counterclockwise, the angle increases in measure. For instance, in the preceding diagram, the terminal arm of the acute angle lies in the first quadrant, whereas the terminal arm of the obtuse angle lies in the second quadrant.

The table of sines and cosines for acute and obtuse angles that you completed in question 1.a. may be used to sketch their graphs. The graphs of $y = \sin\theta$ and $y = \cos\theta$ are as follows:





Notice that for angles between 90° and 180°, sine is positive but cosine is negative.

standard position: the position of an angle drawn in the coordinate plane in which the vertex of the angle is located at the origin and the initial arm of the angle lies along the positive x - axis

initial arm: the arm of an angle in standard position lying along the positive x - axis

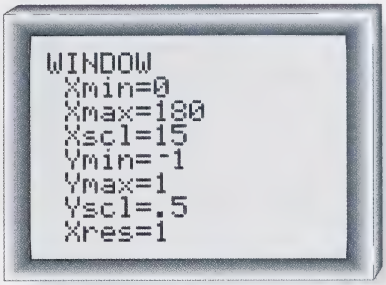
terminal arm: the arm of an angle in standard position that rotates about the x - axis

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

You can use your graphing calculator to obtain these graphs.

Enter the following settings in "WINDOW":

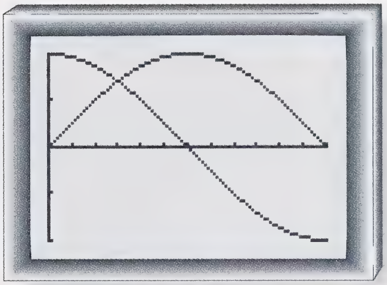


```
WINDOW
  Xmin=0
  Xmax=180
  Xscl=15
  Ymin=-1
  Ymax=1
  Yscl=.5
  Xres=1
```

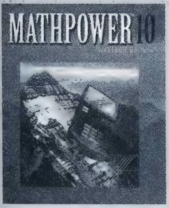
Ensure that you have selected "Degree" in the MODE window.

Enter the sine and cosine functions by pressing the following:

Y= sin X,T,Θ,*n*) ENTER cos X,T,Θ,*n*) GRAPH



You will now look at the definitions of sine and cosine for angles drawn in standard position, and you will investigate why sine is positive and cosine is negative for obtuse angles.

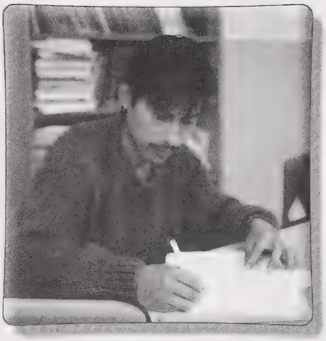


Turn to pages 339 to 341 of *MATHPOWER™ 10* and read from the red line near the top of page 339 to Example 1. Then work through Examples 1, 2, and 4.

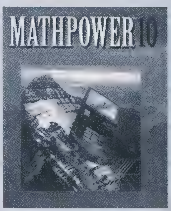
2. Answer the following questions on page 341 of the textbook:

- **a.** questions 1 and 2 of "Practice"
- **b.** questions 6 to 9 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 3.



You have used the definitions for sine and cosine, $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$, and your calculator to find the values of these ratios for any angle between 0° and 180°. In the next exercise, you will reverse the process. You will find the acute and/or obtuse angle from values of the ratios.



Turn to pages 340 to 342 of *MATHPOWER™ 10*. Work through Example 3; read the note at the bottom of page 340; then work through Example 5.

3. Answer the following questions on pages 341 and 342 of the textbook:

a. questions 4 and 5 of "Practice"
b. questions 18 to 20 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 3.

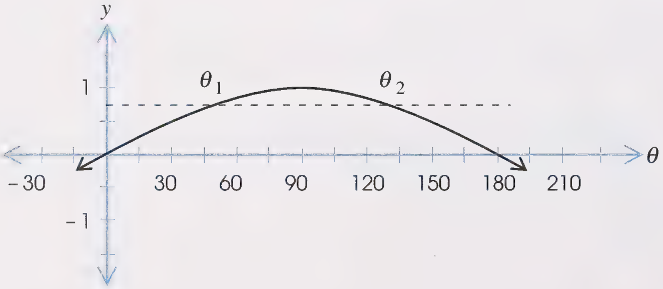
As you have seen, you can find two angles between 0° and 180° given the value of the sine ratio. One of those angles is acute and the other is obtuse—they are supplementary angles. Therefore, the two angles are θ and $180° - \theta$.

$$\sin\theta = \sin(180° - \theta)$$

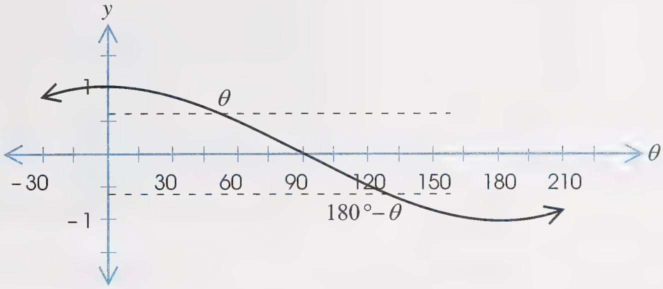
Look at the graph of the sine ratio. Select a *y*-value between 0 and 1. The horizontal line determined by that *y*-value cuts the sine curve at two points for angles between 0° and 180°.

Mathematical Process

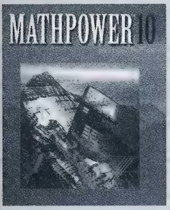
☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☒ Visualization



For the cosine curve, a similar horizontal line would cross the curve at exactly one point between 0° and 180°. However, for two y-values equal in magnitude but opposite in sign, the angles determined are supplementary.



$$\cos\theta = -\cos(180° - \theta) \quad \text{or} \quad \cos(180° - \theta) = -\cos\theta$$



4. Turn to page 342 of *MATHPOWER™ 10* and answer questions 21 to 24 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 3.

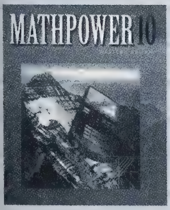
Now Try This

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
☒ Problem Solving
■ Reasoning
■ Technology
■ Visualization

The values of sine and cosine for obtuse angles follow logically from the definitions of sine and cosine for angles drawn in standard position. In your future study of trigonometry, you will investigate the values of sine and cosine for angles of any measure, positive or negative. For instance, you will find the values of $\sin 600°$ or $\cos(-6000°)$. Again, these values will follow logically from the definitions.

The power of logic flows through all branches of mathematics. One concept logically follows from another; logic is the glue that links ideas in mathematics. You will use your logical skills in the next exercise.



5. Turn to page 342 of *MATHPOWER™ 10* and answer the question posed in "LOGIC POWER."

Compare your response with the one in the Appendix, Section 1: Activity 3.

Looking Back

In this activity, you drew acute and obtuse angles in standard position, determined the sine and cosine of any acute and obtuse angle, and found acute and obtuse angles from given values of the sine and cosine ratios.

Many jobs and professions use geometry and trigonometry. The construction trades are good examples. Interview someone who uses trigonometry in his or her work about the use of mathematics in the workplace. Record your interview in your journal.

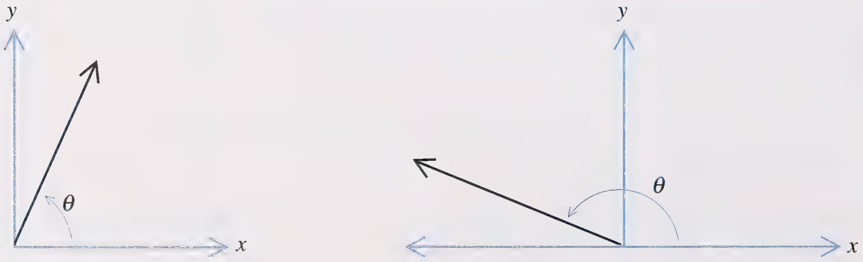
Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you investigated angles in standard position between 0° and 180°. An angle in standard position is one that is drawn on the coordinate plane with the initial arm as the positive *x*-axis, the vertex at the origin, and the terminal arm at some position rotated about the origin.

Angles in Standard Postion



You also discovered that pairs of angles between 0° and 180° have the same sine value and have a cosine values that are equal in magnitude but opposite in sign.

$$\left.\begin{aligned} \sin 30^\circ &= 0.5 \\ \sin 150^\circ &= 0.5 \end{aligned}\right\} \text{Sine values are equal.}$$

$$\left.\begin{aligned} \cos 30^\circ &= 0.8660 \\ \cos 150^\circ &= 0.8660 \end{aligned}\right\} \text{Cosine values are equal in magnitude, but opposite in sign.}$$

1. Write two other pairs of angles between 0° and 180° that have the same sine ratio.

2. Write the rule for determining the sine of the supplement of a given angle.

3. Write two other pairs of angles between 0° and 180° that have the same cosine ratio.

4. Write the rule for determining the cosine of the supplement of a given angle.

5. Evaluate each of the following to 4 decimal places.

- **a.** $\sin 127^\circ$
- **b.** $\cos 11^\circ$
- **c.** $\cos 139^\circ$
- **d.** $\sin 68^\circ$
- **e.** $\cos 168^\circ$

6. Determine the angle of each of the following ratios. Round your answers to the nearest degree.

- **a.** $\sin\theta = 0.2786$
- **b.** $\cos\theta = 0.3940$
- **c.** $\cos\theta = -0.5764$
- **d.** $\sin\theta = 0.9276$

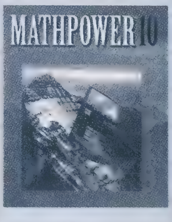
Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

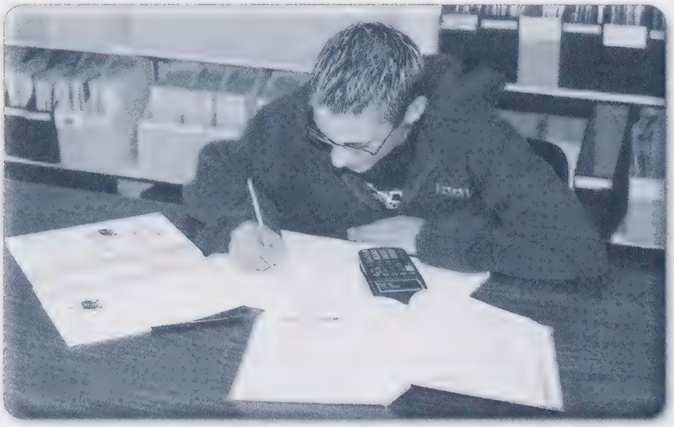
Enrichment

In this section, you found the area of triangles using the formula $A = \frac{1}{2}bh$. The area of a triangle can also be found using Heron's formula.

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is $\frac{1}{2}(a+b+c)$ and a, b, and c are the lengths of the three sides of a triangle

Since applying the formula can be tedious, you can enter a program into your graphing calculator that will calculate the area of a given triangle using Heron's formula. Turn to page 355 of MATHPOWER™ 10 and enter the program for Heron's formula into your graphing calculator.





Answer questions 2 and 3 of "Heron's Formula" on page 355 of the textbook.

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

For more information on Heron's formula, enter the words *Heron's formula* into one of the Internet's search engines and check the sites that are listed.

Conclusion

In this section, you reiewed the trigonometric ratios sine, cosine, and tangent as defined in terms of the lengths of the sides of a right triangle, finding missing sides of angles. You also explored the sine and cosine ratios for acute and obtuse angles drawn in standard position.

Your study of trigonometric ratios and right triangles has provided you with the mathematical tools to determine angles and distances involving right triangles. Many occupations, such as those in the construction industry and surveying industry, require a working knowledge of these ratios and triangles on a daily basis.

Assignment

Turn to Assignment Booklet 5A and complete the assignment for Section 1.

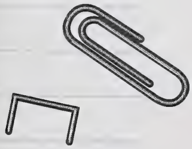
Section 2

The Laws of Sines and Cosines

Do you often see surveyors using a transit along a highway or along a street in town? These surveyors are measuring the angles between lines of sight as well as the distances between various points.

Surveying is an occupation that involves the use of trigonometry. Often, the calculations require the solution of triangles that do not contain right angles.

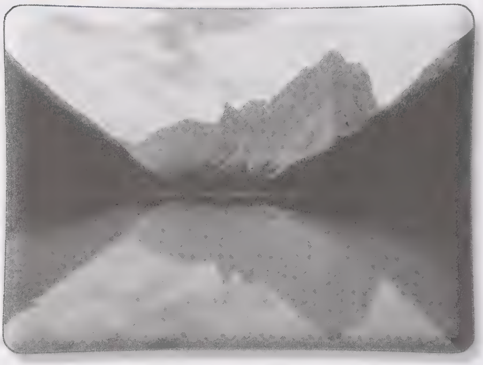
In this section, you will solve triangles that do not contain right angles using the law of sines and the law of cosines.



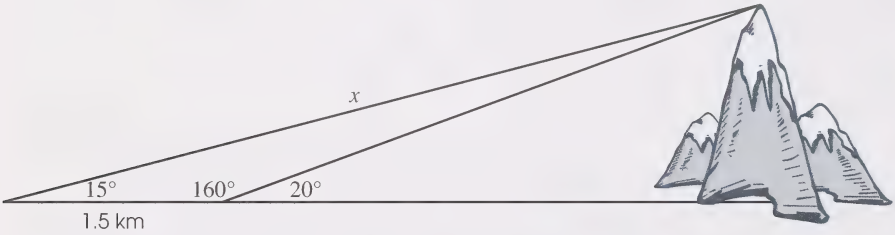
Activity 1: The Law of Sines

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization



While camping on the shore of a lake, Yvanna wishes to determine the distance from her camp to a distant mountain peak (the one reflected in the water) directly across the lake. From her camp, Yvanna calculated the angle of elevation to the top of the mountain to be 15°. Yvanna then jumped into her canoe and proceeded to cross the lake in the direction of the mountain. She noticed that as she got closer to the mountain, the angle of elevation from her position to the top of the mountain increased. Once Yvanna reached the opposite shore, 1.5 km from her camp, she calculated the angle of elevation to the top of the mountain to be 20°.



Now, to determine the distance from her camp to the mountain top, Yvanna will need to know how to solve an **oblique triangle**. An oblique triangle is a triangle that does not contain a right angle.

oblique triangle: a triangle that is not a right triangle

In this activity, you will discover how to solve oblique triangles given two angles and a side.

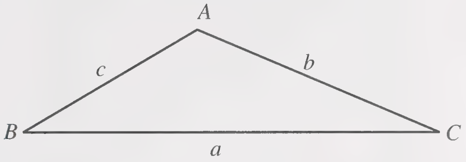
Turn to page 344 of *MATHPOWER*™ *10* and read "The Law of Sines," including "Explore: Interpret the Diagram."

1. Answer questions 1 to 4 of "Inquire" on page 344.

Compare your responses with those in the Appendix, Section 2: Activity 1.

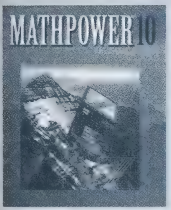
The procedure for solving an oblique triangle given two angles and a side is based on the **law of sines** (or **Sine Law**).

law of sines: the relation stating the lengths of the sides of any triangle are in the same proportion as the sines of the angles opposite the sides



The relationship among the sides and angles of a triangle, as described by the Sine Law, is as follows:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



Turn to pages 344 and 345 of *MATHPOWER™ 10* and work through the development of the law of sines outlined in the text from the red line near the bottom of page 344 to Example 1; then work through Example 1.

2. Answer questions 1, 3, 5, 7, and 9 of "Practice" on page 347 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

When using the law of sines, you must be able to evaluate expressions of the form $\frac{a\sin B}{\sin A}$ on your calculator.

Consider the following example.

Example

Using your graphing calculator, evaluate $\frac{9\sin 40°}{\sin 60°}$.

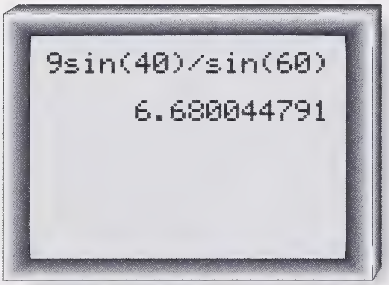
Mathematical Process
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Solution

Press the following:

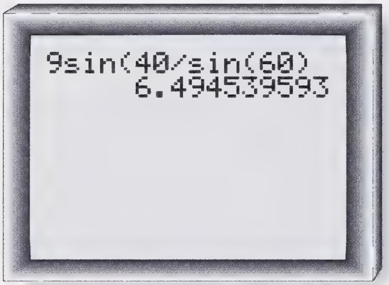
9 sin 4 0) ÷ sin 6 0) ENTER

The following will appear on your display:



```
9sin(40)/sin(60)
      6.680044791
```

It is important that you close off the parentheses for 40°. Otherwise, you will end up multiplying 9 by the sine of, 40 divided by the sine of 60. This will result in the following error:



```
9sin(40/sin(60)
      6.494539593
```

Now, apply what you have discovered about the Sine Law to some problem-solving situations that include areas of oblique triangles.

Turn to pages 346 and 347 of *MATHPOWER™ 10* and work through Examples 2 and 3.

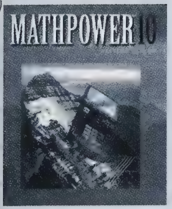
3. Answer the following questions on pages 347 and 348 of the textbook:

  a. questions 11 and 13 of "Practice"
  b. questions 15, 17, and 22.a. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Now Try This

Because the Sine Law was obtained by dividing an oblique triangle into two right triangles, problems that can be solved using the Sine Law can also be solved using a pair of right triangles.



Review Example 2 on pages 334 and 335 of *MATHPOWER™ 10.*

Mathematical Process

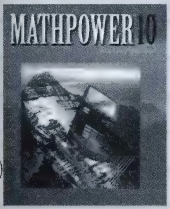
☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☒ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

4. Refer to the question that introduced this activity. Determine the distance, correct to the nearest tenth of a kilometre, from Yvanna's camp to the mountain top using the following:

a. the Sine Law
b. two right triangles

Compare your responses with those in the Appendix, Section 2: Activity 1.

Looking Back



In this activity, you solved oblique triangles given two angles and a side using the law of sines. You also applied the law of sines to find areas and to solve problem situations.

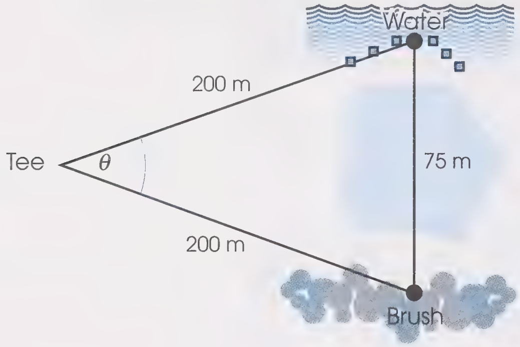
Mathematical Process

☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Now that you have used the law of sines and have investigated using two right triangles to solve similar problems, which method do you find easier? Why? Write your thoughts in your journal.

Activity 2: The Law of Cosines

Many golfers find accuracy a problem when hitting from the tee box. Suppose you intend to drive down a fairway 75 m in width, with water on the left and thick brush on the right. If you consistently drive 200 m, what angle can you drive safely without getting into difficulty?



CORBIS/BOB WINSETT

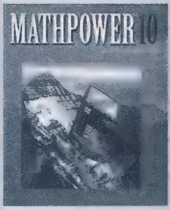
Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

In this activity, you will use the **law of cosines** (or **Cosine Law**) to solve oblique triangles, such as the one created with the golf shot, for missing angles when given the lengths of all three sides. The law of cosines can also be used to solve an oblique triangle when you are given two sides and the angle between those sides.

law of cosines: the relationship between the lengths of the three sides and the cosine of an angle in any triangle

Turn to page 349 of *MATHPOWER™ 10* and read "The Law of Cosines," including "Explore: Use the Diagram."



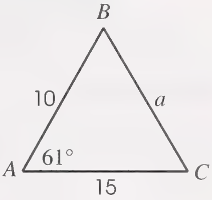
1. Answer questions 1 to 5 of "Inquire" on page 349 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

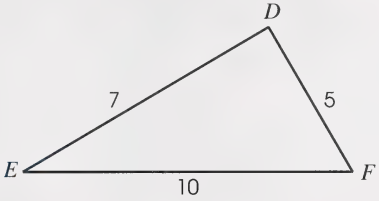
---

contained angle: the angle between two sides of a triangle

Why can't the sine rule be used if you are given three sides or two sides and the **contained angle**? It is because you would end up with two unknowns.



In the triangle given on the right, if you substitute in $\frac{a}{\sin A} = \frac{b}{\sin B}$, the resulting statement, $\frac{a}{\sin 61°} = \frac{15}{\sin B}$, has two unknown terms: a and $\sin B$. Therefore, there is insufficient information given to apply the law of sines.



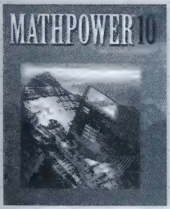
In ΔDEF, again, there is insufficient information given to apply the law of sines.

$$\frac{d}{\sin D} = \frac{e}{\sin E}$$

$$\frac{10}{\sin D} = \frac{5}{\sin E}$$

← This results in two unknowns.

You will now see how to get around these problems using the law of cosines.



Turn to page 350 of *MATHPOWER™ 10* and work through the development of the law of cosines outlined in the text from the top of the page to Example 1; then work through Example 1.

2. Answer questions 1, 3, 7, and 10 of "Practice" on page 352 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Have you noticed how the law of cosines is similar in form to the Pythagorean Theorem? If you are given sides a and b and the contained angle C in ΔABC, you then use the Cosine Law to find side c (the side opposite angle C) as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

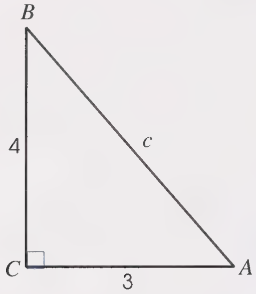
Now look at a right triangle with the two given sides forming the right angle.

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Example

In the triangle given, use the Cosine Law to find side *c*.



Solution

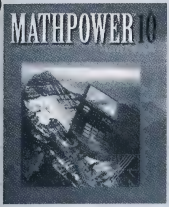
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= a^2 + b^2 - 2ab\cos 90^\circ$$

But, $\cos 90^\circ = 0$. Check that on your calculator!

$$\therefore\ c^2 = a^2 + b^2 - 2ab(0)$$
$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$c = 5$$

The Pythagorean Theorem is simply a special case of the Cosine Law!

Now that you have used the Cosine Law to determine a missing side, you will now investigate how to use the Cosine Law to find missing angles.



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Turn to page 350 of *MATHPOWER™ 10* and read from the red line to the end of page 351, working through Examples 2 and 3.

3. Answer questions 5, 6, 9, and 11 of "Practice" on page 352 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

When determining angles using the Cosine Law, be careful when entering the values into your calculator.

Example

Using your graphing calculator, find the measure of $\angle A$ if $\cos A = \dfrac{5^2+4^2-8^2}{2(5)(4)}$.

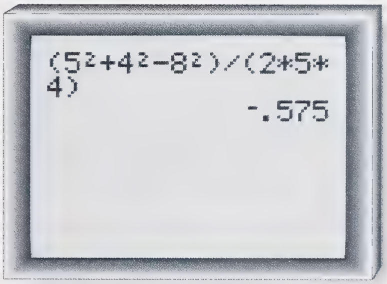
Solution

Method 1

First, evaluate the expression for $\cos A$ by pressing the following:

(5 x^2 + 4 x^2 − 8 x^2)
÷ (2 × 5 × 4) ENTER

The display on your calculator should look as follows:

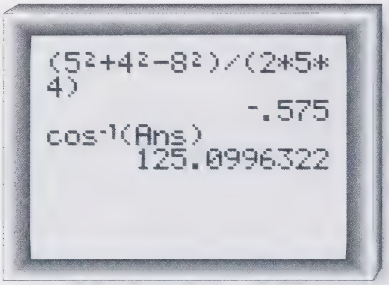


```
(5²+4²-8²)/(2*5*
4)
              -.575
```

Now, to find ∠A, press the following:

2nd [$\cos^{-1}$] 2nd [ANS]) ENTER

The display now gives the measure of ∠A.



```
(5²+4²-8²)/(2*5*
4)
              -.575
cos⁻¹(Ans)
        125.0996322
```

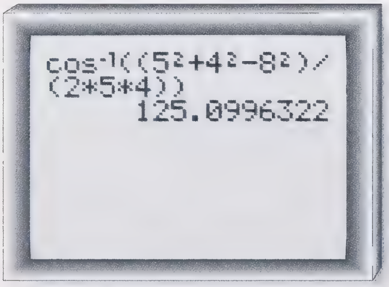
Be careful! Parentheses must be used for both top and bottom. If they were omitted, then division would occur only for $\frac{8^2}{2}$.

Method 2

Instead of doing the calculation in two parts and using the answer from the first part in the second, the value for the angle can be found in a single calculation.

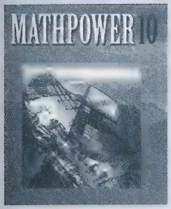
2nd [$\cos^{-1}$] (5 x^2 + 4 x^2 − 8 x^2)
÷ (2 × 5 × 4)) ENTER

The calculator display should look as follows:



```
cos⁻¹((5²+4²-8²)/
(2*5*4))
        125.0996322
```

You will now apply the law of cosines in problem situations.

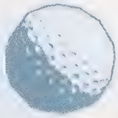


4. Turn to pages 352 and 353 of *MATHPOWER™ 10* and answer questions 15, 17, 20, 23, and 28 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 2: Activity 2.

Now Try This

At the beginning of this activity, you looked at hitting a 200-m drive down a fairway that was only 75 m in width.



5. At the tee box, determine the maximum angle that will keep the golf ball in the fairway. Solve this problem two different ways.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Looking Back

In this activity, you solved oblique triangles using the law of cosines when given any of the following information:

- three sides
- two sides and the contained angle

You also applied what you learned to practical problems.

In your journal, write down formulas for the Sine Law and the Cosine Law. If you were to try and give strategies to your friends that would help them remember these formulas, what would those strategies be? Record your ideas in your journal as well.

Mathematical Process

☒ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

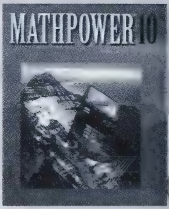
Follow up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

If you have access to the Internet, you can find additional information on the Sine Law and the Cosine Law. Two locations that contain the development of these laws, examples, and extra problems are as follows:

http://www.acts.tinet.ie/sinerule_673.html
http://www.acts.tinet.ie/cosinerule_673.html



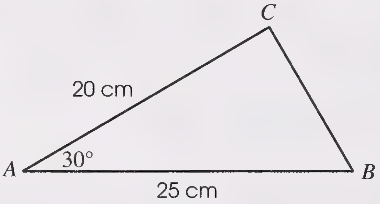
Turn to page 346 of *MATHPOWER™ 10* and work through Example 2. This example describes how the area of an oblique triangle can be found if you are given two angles and a side.

Notice that Example 2 uses the Sine Law together with the area formula to find the area. In Activity 5, the Law of Cosines, you were asked to find areas of triangles when given two sides and the contained angle and when given three sides.

The following examples may help you understand these procedures.

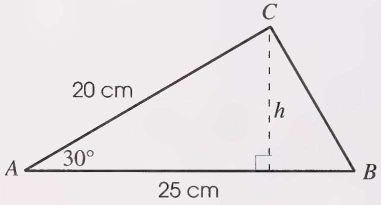
Example

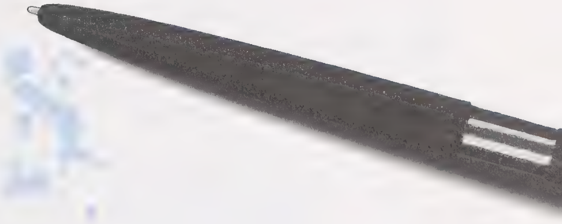
In ΔABC, $\angle A = 30°$, $b = 20$ cm, and $c = 25$ cm. Find the area of the triangle.



Solution

Draw an altitude, h, from C.





Find the length of *h*.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30° = \frac{h}{20}$$

$$h = 20\sin 30°$$

$$= 10 \text{ cm}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}(\text{base})(\text{height})$$

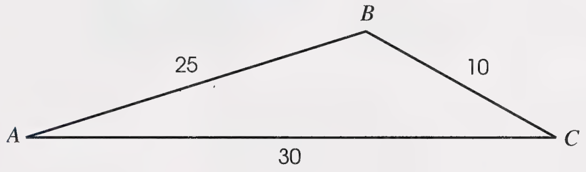
$$= \frac{1}{2}(25)(10)$$

$$= 125 \text{ cm}^2$$

Now, work through an example in which you are given the lengths of three sides.

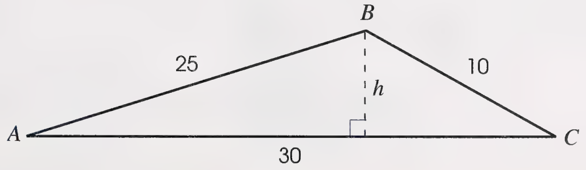
Example

In ΔABC, $a = 10$, $b = 30$ cm, and $c = 25$ cm. Find the area of the triangle to the nearest tenth.



Solution

Draw an altitude, *h*, from *B*.



As in the previous example, if you knew $\angle A$, you could determine the altitude, h, and then use the area formula.

To find $\angle A$, use the Cosine Law.

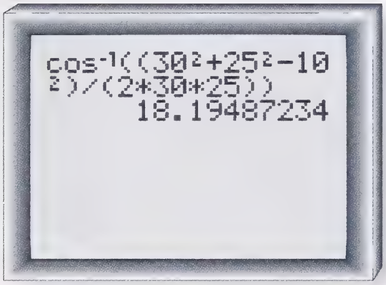
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{30^2 + 25^2 - 10^2}{2(30)(25)}$$

$$\angle A \doteq 18.2°$$

Using your calculator, press the following:

2nd [$\cos^{-1}$] (3 0 x^2 + 2 5 x^2 − 1 0 x^2) ÷ (2 × 3 0 × 2 5)) ENTER



```
cos⁻¹((30²+25²-10
²)/(2*30*25))
     18.19487234
```

Hint: *Leave the answer on your display; you will use it to find h.*

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Find the length of h.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

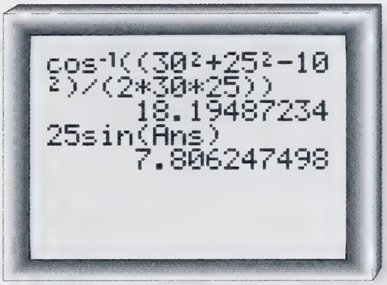
$$\sin A = \frac{h}{25}$$

$$h = 25\sin A$$

Mathematical Process
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Using your calculator, press the following:

2 5 sin 2nd [ANS]) ENTER



```
cos⁻¹((30²+25²-10
²)/(2*30*25))
          18.19487234
25sin(Ans)
          7.806247498
```

Hint: Leave this value of h on your display.

$\therefore\ h \doteq 7.8\text{ cm}$

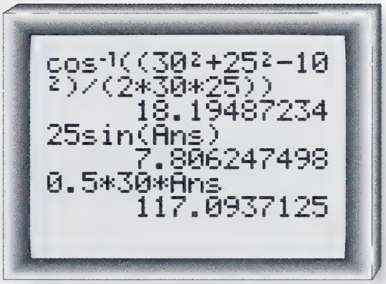
Finally, find the area of the triangle.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bh\\ &= \frac{1}{2}(30)h\end{aligned}$$

Mathematical Process
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Using your calculator, press the following:

0 • 5 × 3 0 × 2nd [ANS] ENTER



```
cos⁻¹((30²+25²-10
²)/(2*30*25))
          18.19487234
25sin(Ans)
          7.806247498
0.5*30*Ans
          117.0937125
```

The area of ΔABC is about 117.1 cm^2.

Now, you try to find these areas on your own!

1. Find the area of ΔABC given $\angle A = 150°$, $b = 10$, and $c = 18$.

2. Find the area of ΔABC given $a = 18$, $b = 10$, and $c = 12$ to the nearest tenth.

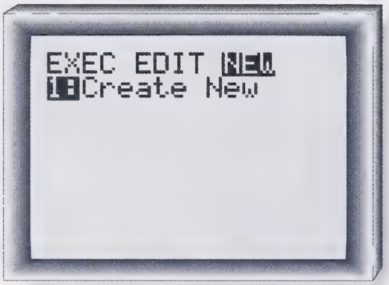
Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.

Enrichment

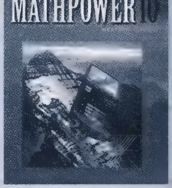
In Module 2, you programmed your calculator to determine the distance between two points in the plane and the midpoint of a line segment. Now that you have worked with the Sine Law and the Cosine Law, you should be able to program your calculator to solve oblique triangles for missing sides or angles.

Turn to "TECHNOLOGY" on page 354 of *MATHPOWER™ 10* and enter the program for the law of sines given at the top of the page.

Begin by opening the program window by pressing PRGM. Select the NEW menu using the arrow keys.



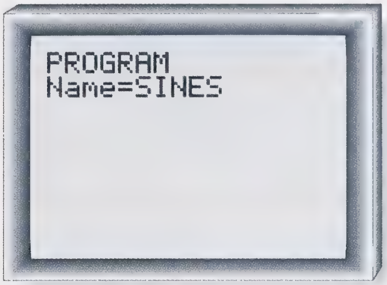
Press ENTER.



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Now, the calculator prompts you to name the program. Name the program "SINES."



```
PROGRAM
Name=SINES
```

Now, enter the lines of the program (as given on page 354 of *MATHPOWER™ 10*) by pressing the following:

PRGM [Select the I/O menu.] 8 (8:ClrHome) ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA ["] ALPHA [B] 2nd
[TEST] 1 (1:=) ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 (1:Input) ALPHA [B] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [X] 2nd [TEST] 1
ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 ALPHA [X] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [Y] 2nd [TEST] 1
ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 ALPHA [Y] ENTER

ALPHA [B] sin ALPHA [Y]) ÷ sin ALPHA [X]) STO→
ALPHA [A] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [A] 2nd [TEST] 1
ALPHA ["] ENTER

PRGM [Select the I/O menu.] 3 ALPHA [A] 2nd [QUIT]

To run the program, press PRGM and select "SINES" under the EXEC menu.

1. Turn to page 354 of *MATHPOWER™ 10* and answer question 2 of Investigation 1, “The Law of Sines”.
2. Now program the law of cosines into your graphing calculator. Then, turn to pages 354 and 355 of *MATHPOWER™ 10* and answer questions 1 to 4 of Investigation 2, “The Law of Cosines.”

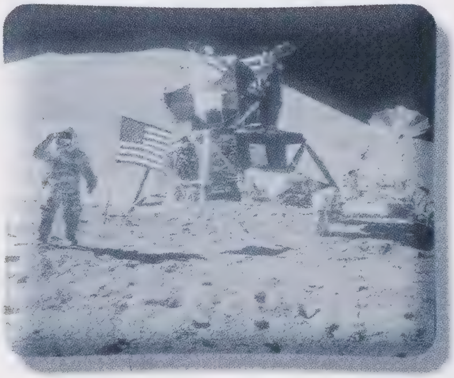
Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section, you developed and applied two laws for solving oblique triangles: the law of sines and the law of cosines.

Remember, trigonometry means *the measurement of triangles*. Having completed this section, you can now measure oblique triangles.

The need to measure triangles developed along with the need to survey land, lay out construction sites, and determine the distances of inaccessible objects. Trigonometry is still one of the essential tools of the modern surveyor, construction engineer, and mapmaker. But, trigonometry is not limited to these applications; it is also fundamental to modern science. The footsteps in the lunar dust can be attributed in no small part to trigonometry.



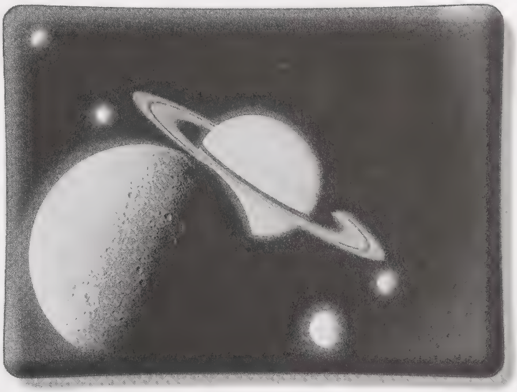
NASA

Assignment

Turn to Assignment Booklet 5B and complete the assignment for Section 2.

Module Summary

In Section 1, similar right triangles laid the basis for your study of trigonometry. No longer restricted to the right triangle, you extended the definitions of sine and cosine to any angle drawn in standard position (between 0° and 180°). Once you were confident with these definitions, you worked with the Sine Law and the Cosine Law in Section 2 to solve oblique triangles.



JPL

Trigonometry provides mathematical tools used to describe the everyday world. Not only will trigonometry answer the questions "How big?" and "How far?" in the world today, it will also answer the universal questions of tomorrow. Who has not dreamed of setting foot on a distant planet, thanks in no small part to the union of mathematics and technology?

Final Module Assignment

Turn to Assignment Booklet 5B and complete the final module assignment.

COURSE SURVEY FOR PURE MATHEMATICS 10
(© 2002)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name ______________________

Age ☐ under 19 ☐ 19 to 40 ☐ over 40

Address ______________________

____

____

File No. ______________________

Date ______________________

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

____

____

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

☐ Yes ☐ No If yes, which style do you prefer?

____

____

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

☐ Yes ☐ No If yes, explain.

____

____

4. Were the questions and directions easy to understand?

☐ Yes ☐ No If no, explain.

____

____

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ☐ mainly extra help
   ☐ mainly enrichment
   ☐ a variety
   ☐ none

   Did you find these activities beneficial?

   ☐ Yes ☐ No If no, explain.

6. Did you understand what was expected in the Assignment Booklets?

   ☐ Yes ☐ No If no, explain.

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ☐ Yes ☐ No If no, provide details.

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ☐ Yes ☐ No Comment on the lines below.

   b. Did you find the variety of visuals in this course motivating?

      ☐ Yes ☐ No Comment on the lines below.

9. Some activities may have called for the use of an audiocassette, videocassette, or CD. Did you use these forms of media?

   ☐ Yes ☐ No Comment on the lines below.

Only students enrolled in a junior high course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your teacher. How well did you work as a team?

    Student's comments:

    Teacher's comments:

Course Content

1. Was enough detailed information provided to help you learn the expected skills and objectives?

   ☐ Yes ☐ No Comment on the lines below.

2. Did you find the workload reasonable?

   ☐ Yes ☐ No If no, explain.

3. Did you have any difficulty with the reading level?

☐ Yes ☐ No Please comment.

4. How would you assess your general reading level?

☐ poor reader ☐ average reader ☐ good reader

5. Was the material presented clearly and with sufficient depth?

☐ Yes ☐ No If no, explain.

General

1. What did you like least about the course?

2. What did you like most about the course?

Additional Comments

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ☐ Yes ☐ No If yes, approximately how many times? ______

   Did you find the staff helpful?

   ☐ Yes ☐ No If no, explain.

2. Were you able to fax any of your assignment response pages?

   ☐ Yes ☐ No If yes, comment on the value of being able to do this.

3. If you mailed your assignment response pages, how long did it take for their return?

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ☐ Yes ☐ No Please comment.

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

If you are enrolled at the Alberta Distance Learning Centre and will be mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Appendix

Glossary
Suggested Answers

Glossary

angle of depression: the angle through which an observer looks downward from the horizontal to see an object below

angle of elevation: the angle through which an observer looks upward from the horizontal to see an object above

contained angle: the angle between two sides of a triangle

initial arm: the arm of an angle in standard position lying along the positive x-axis

law of cosines (or Cosine Law): the relationship between the lengths of the three sides and the cosine of an angle in any triangle

$$a^2 = b^2 + c^2 - 2bc\cos A$$

law of sines (or Sine Law): the relation stating the lengths of the sides of any triangle are in the same proportion as the sines of the angles opposite the sides

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

oblique triangle: a triangle that is not a right triangle

solving a right triangle: determining the lengths of its sides and the measures of its angles

standard position: the position of an angle drawn in the coordinate plane in which the vertex of the angle is located at the origin and the initial arm of the angle lies along the positive x-axis

terminal arm: the arm of an angle in standard position that rotates about the x-axis

Section 1: Activity 1

1. a. **Textbook questions 1 and 2 of Investigation 1, "Sine, Cosine, and Tangent," p. 328**

1. $\sin B = \frac{4}{5}$, $\cos B = \frac{3}{5}$, and $\tan B = \frac{4}{3}$

2. a. $\sin D = \frac{12}{13}$, $\cos D = \frac{5}{13}$, and $\tan D = \frac{12}{5}$

$\sin F = \frac{5}{13}$, $\cos F = \frac{12}{13}$, and $\tan F = \frac{5}{12}$

b. $\sin J = \frac{4}{2\sqrt{5}}$, $\cos J = \frac{2}{2\sqrt{5}}$, and $\tan J = \frac{4}{2}$

$= \frac{2}{\sqrt{5}}$ $\quad = \frac{1}{\sqrt{5}}$ $\quad = 2$

$\sin L = \frac{2}{2\sqrt{5}}$, $\cos L = \frac{4}{2\sqrt{5}}$, and $\tan L = \frac{2}{4}$

$= \frac{1}{\sqrt{5}}$ $\quad = \frac{2}{\sqrt{5}}$ $\quad = \frac{1}{2}$

b. Textbook questions 1 to 3 of Investigation 2, "Special Right Triangles," p. 328

1. **a.** $\sin 45° = \frac{1}{\sqrt{2}}$ **b.** $\cos 45° = \frac{1}{\sqrt{2}}$ **c.** $\tan 45° = 1$

2. **a.** $\sin 30° = \frac{1}{2}$ **b.** $\cos 30° = \frac{\sqrt{3}}{2}$ **c.** $\tan 30° = \frac{1}{\sqrt{3}}$

3. **a.** $\sin 60° = \frac{\sqrt{3}}{2}$ **b.** $\cos 60° = \frac{1}{2}$ **c.** $\tan 60° = \sqrt{3}$

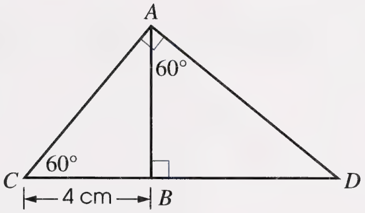
c. Textbook questions 1 and 2 of Investigation 3, "Problems Involving Two Special Right Triangles," p. 329

1. From ΔABC, $\tan C = \frac{AB}{BC}$

$$\tan 60° = \frac{AB}{4}$$

$$\sqrt{3} = \frac{AB}{4}$$

$$AB = 4\sqrt{3} \text{ cm}$$



From ΔABC, $\cos C = \frac{BC}{AC}$

$$\cos 60° = \frac{4}{AC}$$

$$\frac{1}{2} = \frac{4}{AC}$$

$$AC = 8 \text{ cm}$$

From ΔABD, $\tan A = \frac{BD}{AB}$

$$\tan 60° = \frac{BD}{4\sqrt{3}}$$

$$\sqrt{3} = \frac{BD}{4\sqrt{3}}$$

$$BD = (4\sqrt{3})(\sqrt{3})$$

$$= 4(3)$$

$$= 12 \text{ cm}$$

From ΔABD, $\cos A = \frac{AB}{AD}$

$$\cos 60° = \frac{4\sqrt{3}}{AD}$$

$$\frac{1}{2} = \frac{4\sqrt{3}}{AD}$$

$$AD = 2(4\sqrt{3})$$

$$= 8\sqrt{3} \text{ cm}$$

Section 1: Activity 1 (continued)

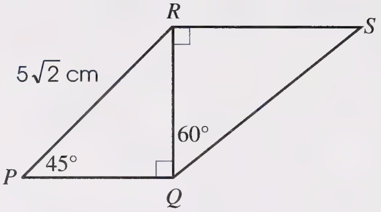
2. From ΔPQR, $\cos P = \dfrac{PQ}{PR}$

$$\cos 45° = \frac{PQ}{5\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{PQ}{5\sqrt{2}}$$

$$PQ = \frac{5\sqrt{2}}{\sqrt{2}}$$

$$= 5 \text{ cm}$$



From ΔPQR, $QR = PQ$

$$= 5 \text{ cm}$$

From ΔQRS, $\cos Q = \dfrac{QR}{QS}$

$$\cos 60° = \frac{5}{QS}$$

$$\frac{1}{2} = \frac{5}{QS}$$

$$QS = 5(2)$$

$$= 10 \text{ cm}$$

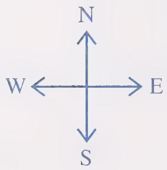
From ΔQRS, $\tan Q = \dfrac{RS}{QR}$

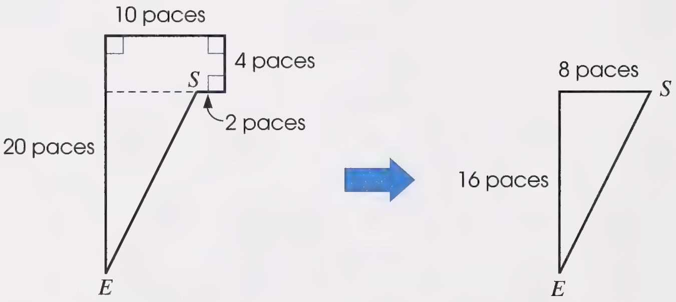
$$\tan 60° = \frac{RS}{5}$$

$$\sqrt{3} = \frac{RS}{5}$$

$$RS = 5\sqrt{3} \text{ cm}$$

2. a. Textbook question "Explore: Draw a Diagram," p. 330





b. Textbook questions 1 to 6 of "Inquire," p. 330

1. The legs are 16 paces and 8 paces in length.

2. The tangent ratio would be used to find $\angle E$.

$$\begin{aligned}\tan E &= \frac{\text{opp}}{\text{adj}}\\ &= \frac{8}{16}\\ &= \frac{1}{2}\\ &= 0.5\end{aligned}$$

3. $\tan E = 0.5$

$\angle E \doteq 27°$

Note: *Make sure your calculator is in the degree mode.*

4. You can find the length of ES by using the Pythagorean Theorem or by using the sine or cosine ratios and the measure of $\angle E$.

5. Using the Pythagorean Theorem,

$$\begin{aligned}ES^2 &= 16^2 + 8^2\\ ES^2 &= 256 + 64\\ ES^2 &= 320\\ ES &= \sqrt{320}\\ &\doteq 18 \text{ paces}\end{aligned}$$

6. Holmes could go directly from E to S by walking about 18 paces at an angle of approximately 27° east of north.

3. a. Textbook questions 1 and 2 of Investigation 4, "Trigonometric Ratios of Any Acute Angle," p. 329

1.
- **a.** $\sin 27° \doteq 0.454$
- **b.** $\cos 56° \doteq 0.559$
- **c.** $\tan 78° \doteq 4.705$
- **d.** $\cos 7° \doteq 0.993$
- **e.** $\tan 40° \doteq 0.839$
- **f.** $\sin 62° \doteq 0.883$

2.
- **a.** $\sin D = 0.602$, $\angle D \doteq 37°$
- **b.** $\cos A = 0.309$, $\angle A \doteq 72°$
- **c.** $\tan C = 0.445$, $\angle C \doteq 24°$
- **d.** $\tan R = 2.246$, $\angle R \doteq 66°$
- **e.** $\sin X = 0.978$, $\angle X \doteq 78°$
- **f.** $\cos W = 0.951$, $\angle W \doteq 18°$

Section 1: Activity 1 (continued)

b. Textbook questions 1, 3, and 6 of Investigation 5, "Finding Side Lengths in Any Right Triangle," p. 329

1. $\sin 23° = \frac{x}{20}$

$x = 20 \sin 23°$

$\doteq 7.8 \text{ cm}$

3. $\tan 42° = \frac{x}{12}$

$x = 12 \tan 42°$

$\doteq 10.8 \text{ m}$

6. $\cos 65° = \frac{x}{40}$

$x = 40 \cos 65°$

$\doteq 16.9 \text{ m}$

c. Textbook questions 1 to 3 of Investigation 6, "Finding Angles in Any Right Triangle," p. 329

1. $\sin x = \frac{5}{8}$

$\sin x = 0.625$

$\angle x \doteq 39°$

2. $\cos x = \frac{9}{14}$

$\cos x \doteq 0.6429$

$\angle x \doteq 50°$

3. $\tan x = \frac{6}{8}$

$\tan x = 0.75$

$\angle x \doteq 37°$

4. a. Textbook questions 2, 4, 6, and 8 of "Practice," p. 332

2. $\angle F = 90° - 41°$

$= 49°$

$\cos 41° = \frac{60}{d}$

$d \cos 41° = 60$

$d = \frac{60}{\cos 41°}$

$\doteq 80 \text{ m}$

$\tan 41° = \frac{e}{60}$

$e = 60 \tan 41°$

$\doteq 52 \text{ m}$

4. $q^2 = 8^2 + 13^2$

$q^2 = 64 + 169$

$q^2 = 233$

$q = \sqrt{233}$

$\doteq 15 \text{ cm}$

$\tan R = \frac{13}{8}$

$\angle R \doteq 58°$

$\angle P \doteq 90° - 58°$

$\doteq 32°$

6. $\angle M = 90° - 57.4°$

$= 32.6°$

$\tan 57.4° = \frac{20.3}{m}$

$m \tan 57.4° = 20.3$

$m = \frac{20.3}{\tan 57.4°}$

$\doteq 13.0 \text{ m}$

$\sin 57.4° = \frac{20.3}{n}$

$n \sin 57.4° = 20.3$

$n = \frac{20.3}{\sin 57.4°}$

$\doteq 24.1 \text{ m}$

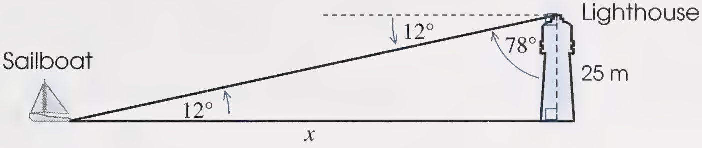
8. $j^2 = 68.8^2 + 72.3^2$ $\quad$ $\tan K = \frac{68.8}{72.3}$ $\quad$ $\angle L \doteq 90° - 43.6°$

$$j = \sqrt{68.8^2 + 72.3^2} \qquad \angle K \doteq 43.6° \qquad \doteq 46.4°$$

$$\doteq 99.8 \text{ cm}$$

b. Textbook questions 9, 10, 11, 15, and 17 of "Applications and Problem Solving," p. 332

9.



Let x represent the distance from the sailboat to the base of the lighthouse.

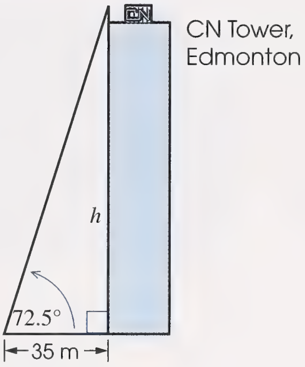
$$\therefore \tan 78° = \frac{x}{25}$$
$$x = 25 \tan 78°$$
$$\doteq 118$$

The sailboat is approximately 118 m from the lighthouse.

10. **a.** Let h be the height of the CN Tower in Edmonton.

$$\therefore \tan 72.5 = \frac{h}{35}$$
$$h = 35 \tan 72.5$$
$$\doteq 111$$

The height of Edmonton's CN Tower is about 111 m.



b. $$\frac{\text{Toronto's CN Tower}}{\text{Edmonton's CN Tower}} \doteq \frac{555}{111}$$
$$\doteq 5$$

The CN Tower in Toronto is about 5 times taller than the CN Tower in Edmonton.

Section 1: Activity 1 (continued)

c. Let r represent the number of storeys in Toronto's CN Tower.

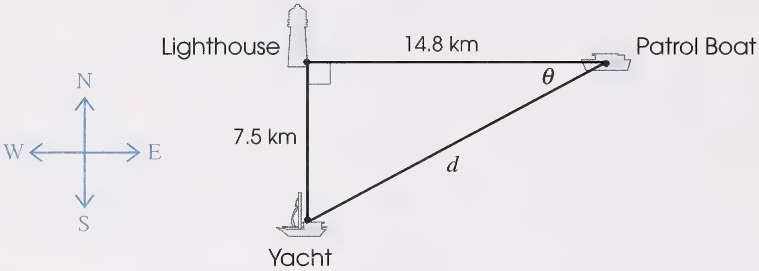
$$\therefore \frac{n}{27} \doteq 5$$

$$n \doteq 5(27)$$

$$\doteq 135$$

You would expect the CN Tower in Toronto to have about 135 storeys.

11. a.



Let d represent the distance between the patrol boat and the disabled yacht.

$$d^2 = 7.5^2 + 14.8^2$$

$$d = \sqrt{7.5^2 + 14.8^2}$$

$$\doteq 16.6$$

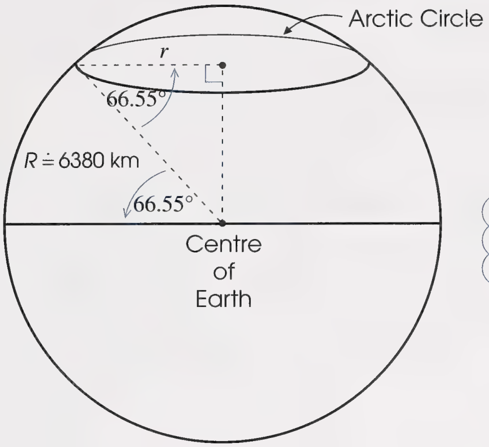
The distance to the yacht is approximately 16.6 km.

b. $\tan\theta = \dfrac{7.5}{14.8}$

$$\theta \doteq 26.9°$$

The patrol boat should head approximately 26.9° south of west in order to reach the disabled yacht.

15.



Note: R represents the radius of Earth, which is approximately 6380 km.

Let r represent the radius of the Arctic Circle.

$$\therefore \cos 66.55° \doteq \frac{r}{6380}$$

$$r \doteq 6380 \cos 66.55°$$

$$\begin{aligned}\therefore C &= 2\pi r \\ &= 2\pi(6380 \cos 66.55°) \\ &\doteq 15\,952\end{aligned}$$

The length of the Arctic Circle is about 15 952 km.

17.

	Angle x			Angle $(90° - x)$		
Triangle	**tan**	**sin**	**cos**	**tan**	**sin**	**cos**
ABC	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{3}{5}$
DEF	$\frac{5}{12}$	$\frac{5}{13}$	$\frac{12}{13}$	$\frac{12}{5}$	$\frac{12}{13}$	$\frac{5}{13}$

a. $\tan x$ is the reciprocal of $\tan(90° - x)$.

b. $\sin x = \cos(90° - x)$

c. $\cos x = \sin(90° - x)$

d. If two angles are complementary, where the sum of their measures is 90°, then

- the tangents of the angles are reciprocals
- the sine of one angle equals the cosine of the other angle

Section 1: Activity 1 (continued)

5. **Textbook questions a. to d. of "LOGIC POWER," p. 337**

   a. Eight cubes have 3 blue faces; they are the cubes at the corners.

   b. There are 12 cubes with 2 blue faces, one on each of the 12 edges of the cube.

   c. There are 6 cubes with a single blue face; there is one of these cubes on each face of the large cube.

   d. Because there are 5^3 or 125 small cubes in the large cube, there must be $125-(8+12+6)=99$ small cubes with 0 blue faces.

Section 1: Activity 2

1. **Textbook questions 1 to 3 of "Inquire," p. 333**

   1. a. $\tan BAC=\dfrac{BC}{AB}$

   $BC=AB\tan BAC$

   b. $BC=25\tan 74.8°$

   c. $BC \doteq 92.0\ \text{m}$

   2. a. $\tan BAD=\dfrac{BD}{AB}$

   $BD=AB\tan BAD$

   b. $BD=25\tan 76.3°$

   c. $BD\doteq 102.6\ \text{m}$

   3. $$\begin{aligned}CD&=BD-BC\\&\doteq 102.6-92.0\\&\doteq 10.6\end{aligned}$$

   The height of the mast is approximately 10.6 m.

2. **Textbook questions 2, 4, 6, and 8 of "Practice," pp. 335 and 336**

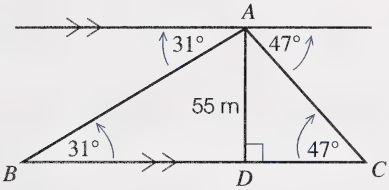
   2. In ΔDEG, $\cos 56.1°=\dfrac{150.5}{EG}$

   $$EG=\frac{150.5}{\cos 56.1°}$$

   In ΔEFG, $\cos 35.7°=\dfrac{EG}{EF}$

   $$\begin{aligned}EF&=\frac{EG}{\cos 35.7°}\\&=\frac{\frac{150.5}{\cos 56.1°}}{\cos 35.7°}\\&=\frac{150.5}{(\cos 56.1°)(\cos 35.7°)}\\&\doteq 332.3\ \text{m}\end{aligned}$$

4. Because the line through A is parallel to BC, $\angle B = 31°$ and $\angle C = 47°$.



In ΔABD, $\tan 31° = \dfrac{55}{BD}$

$$BD = \frac{55}{\tan 31°}$$

In ΔACD, $\tan 47° = \dfrac{55}{CD}$

$$CD = \frac{55}{\tan 47°}$$

$$\begin{aligned} \therefore BC &= BD + CD \\ &= \frac{55}{\tan 31°} + \frac{55}{\tan 47°} \\ &\doteq 143 \text{ m} \end{aligned}$$

6. In ΔPOR, $\tan(21° + 36°) = \dfrac{PR}{60}$

$$PR = 60 \tan 57°$$

In ΔQOR, $\tan 36° = \dfrac{QR}{60}$

$$QR = 60 \tan 36°$$

$$\begin{aligned} \therefore PQ &= PR - QR \\ &= 60 \tan 57° - 60 \tan 36° \\ &\doteq 49 \text{ m} \end{aligned}$$

8. In ΔTUW, $\tan 38.8° = \dfrac{TU}{UW}$

$$UW = \frac{TU}{\tan 38.8°}$$

In ΔTUV, $\tan 53.8° = \dfrac{TU}{UV}$

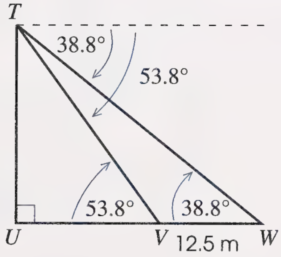
$$UV = \frac{TU}{\tan 53.8°}$$

$$\therefore UW - UV = VW$$

$$\frac{TU}{\tan 38.8°} - \frac{TU}{\tan 53.8°} = 12.5$$

$$TU\left(\frac{1}{\tan 38.8°} - \frac{1}{\tan 53.8°}\right) = 12.5$$

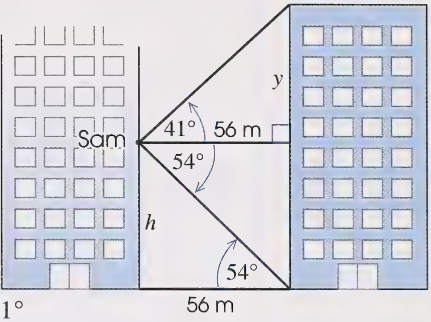
$$\begin{aligned} TU &= \frac{12.5}{\left(\frac{1}{\tan 38.8°} - \frac{1}{\tan 53.8°}\right)} \\ &\doteq 24.4 \text{ m} \end{aligned}$$



Section 1: Activity 2 (continued)

3. Textbook questions 9, 11, 12, 14, and 18 of "Applications and Problem Solving," pp. 336 and 337

9. a. Let h represent the height Sam is above the ground. Therefore, the total height of the second building is $h+y$.



$$\tan 54° = \frac{h}{56} \qquad \tan 41° = \frac{y}{56}$$

$$h = 56\tan 54° \qquad y = 56\tan 41°$$

$$\begin{aligned}\therefore \text{ Height of the second building} &= h+y \\ &= 56\tan 54° + 56\tan 41° \\ &\doteq 126\end{aligned}$$

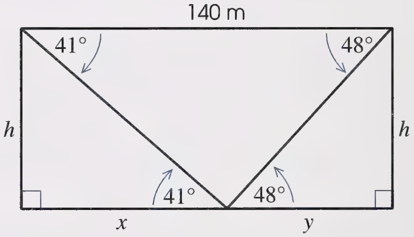
The second building is approximately 126 m tall.

b. $h = 56\tan 54°$

$\doteq 77$

Sam is approximately 77 m above the ground.

11. Let h represent the height of the bridge. Also, $x+y=140$ m.



$$\tan 41° = \frac{h}{x} \qquad \tan 48° = \frac{h}{y}$$

$$h = x\tan 41° \qquad h = y\tan 48°$$

$$x = \frac{h}{\tan 41°} \qquad y = \frac{h}{\tan 48°}$$

$$\begin{aligned}\therefore x+y &= 140 \\ \frac{h}{\tan 41°} + \frac{h}{\tan 48°} &= 140 \\ h\left(\frac{1}{\tan 41°} + \frac{1}{\tan 48°}\right) &= 140 \\ h &= \frac{140}{\left(\frac{1}{\tan 41°} + \frac{1}{\tan 48°}\right)} \\ &\doteq 68\end{aligned}$$

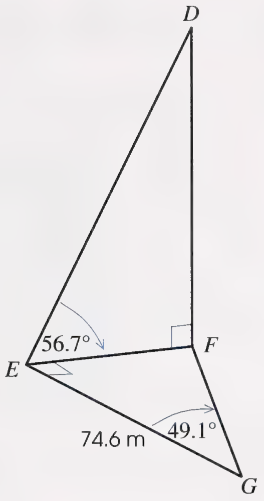
The height of the bridge is approximately 68 m.

12. **a.** In ΔEFG, $\tan 49.1° = \dfrac{EF}{74.6}$

$$EF = 74.6 \tan 49.1°$$

In ΔDEF, $\tan 56.7° = \dfrac{DF}{EF}$

$$\begin{aligned} DF &= EF \tan 56.7° \\ &= (74.6 \tan 49.1°)(\tan 56.7°) \\ &\doteq 131.1 \end{aligned}$$



DF is approximately 131.1 m.

b. $DF + 1.6 \doteq 131.1 + 1.6$

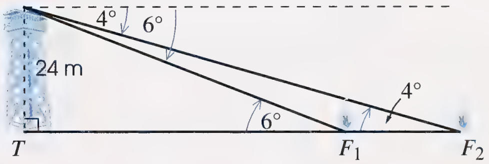
$$\doteq 132.7$$

The cliff is approximately 132.7 m tall.

14. **a.** $\tan 6° = \dfrac{24}{F_1T}$ $\qquad$ $\tan 4° = \dfrac{24}{F_2T}$

$F_1T = \dfrac{24}{\tan 6°}$ $\qquad$ $F_2T = \dfrac{24}{\tan 4°}$

$$\begin{aligned} \therefore\ F_1F_2 &= F_2T - F_1T \\ &= \frac{24}{\tan 4°} - \frac{24}{\tan 6°} \\ &\doteq 115 \end{aligned}$$

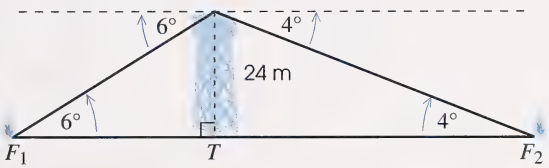


The forest fires are about 115 m apart.

b. $\tan 6° = \dfrac{24}{F_1T}$ $\qquad$ $\tan 4° = \dfrac{24}{F_2T}$

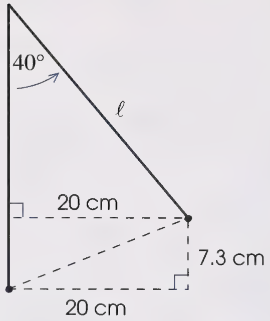
$F_1T = \dfrac{24}{\tan 6°}$ $\qquad$ $F_2T = \dfrac{24}{\tan 4°}$

$$\begin{aligned} \therefore\ F_1F_2 &= F_1T + F_2T \\ &= \frac{24}{\tan 6°} + \frac{24}{\tan 4°} \\ &\doteq 572 \end{aligned}$$



The two forest fires are approximately 572 m apart.

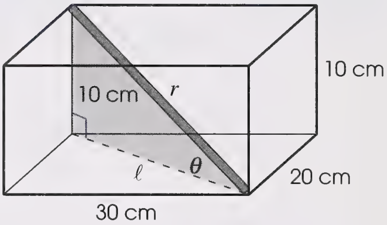
Section 1: Activity 2 (continued)



18. Let ℓ represent the length of the pendulum.

$$\sin 40° = \frac{20}{\ell}$$
$$\ell = \frac{20}{\sin 40°}$$
$$\doteq 31$$

The pendulum is approximately 31 cm long.



4. Let r represent the length of the rod, ℓ represent the base of the right triangle in which r is the hypotenuse, and θ represent the angle the rod makes with the bottom of the box.

First, find ℓ.

$$\ell^2 = 20^2 + 30^2$$
$$\ell^2 = 400 + 900$$
$$\ell^2 = 1300$$
$$\ell = \sqrt{1300}$$

Next, find r using the Pythagorean Theorem.

$$r^2 = 10^2 + \left(\sqrt{1300}\right)^2$$
$$r^2 = 100 + 1300$$
$$r^2 = 1400$$
$$r \doteq 37.4$$

The longest rod that would fit in the box is approximately 37.4 cm in length.

Lastly, find θ.

$$\tan\theta = \frac{10}{\ell}$$
$$\tan\theta = \frac{10}{\sqrt{1300}}$$
$$\theta \doteq 15.5°$$

The angle between the bottom of the box and the rod is about 15.5°.

Section 1: Activity 3

1. **a. Textbook question "Explore: Look for Patterns," p. 338**

Angle A	$\sin A$	$\cos A$
0°	0	1
15°	0.2588	0.9659
30°	0.5	0.8660
45°	0.7071	0.7071
60°	0.8660	0.50
75°	0.9659	0.2588
90°	1	0
105°	0.9659	−0.2588
120°	0.8660	−0.5
135°	0.7071	−0.7071
150°	0.5	−0.8660
165°	0.2588	−0.9659
180°	0	−1

b. Textbook questions 1 to 7 of "Inquire," pp. 338 and 339

1. The sine ratio increases from 0 to 1 between 0° and 90°. The sine ratio decreases from 1 to 0 between 90° and 180°.

2. a. $\sin 0^\circ = \sin 180^\circ$

$\sin 15^\circ = \sin 165^\circ$

$\sin 30^\circ = \sin 150^\circ$

$\sin 45^\circ = \sin 135^\circ$

$\sin 60^\circ = \sin 120^\circ$

$\sin 75^\circ = \sin 105^\circ$

b. The angles in each pair are supplementary—the sum of the angles is 180°.

3. If $\sin A = \sin 40^\circ$, then $\angle A = 40^\circ$ or 140°.

4. If $\sin B = \sin 130^\circ$, then $\angle B = 130^\circ$ or 50°.

5. The cosine ratio decreases from 1 to 0 between 0° and 90°. The cosine ratio continues to decrease from 0 to −1 between 90° and 180°.

6. a. $\cos 30^\circ = 0.8660$ and $\cos 150^\circ = -0.8660$

The $\cos 30^\circ$ and $\cos 150^\circ$ are equal in magnitude but opposite in sign.

b. $\cos 165^\circ = -0.9659$ and $\cos 15^\circ = 0.9659$

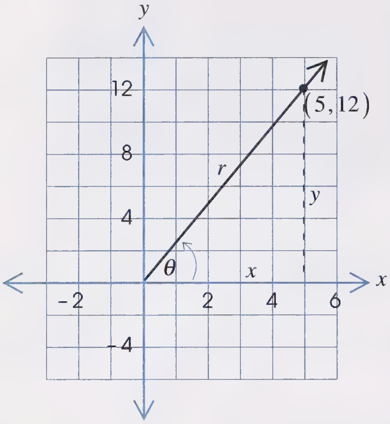
The $\cos 165^\circ$ and $\cos 15^\circ$ are equal in magnitude but opposite in sign.

7. a. The angle made by the clapper board is approximately 15°.
b. $\sin 15^\circ = \sin 165^\circ$
c. No, the cosine of an obtuse angle is negative, whereas the cosine of an acute angle is positive.

Section 1: Activity 3 (continued)

2. a. Textbook questions 1 and 2 of "Practice," p. 341

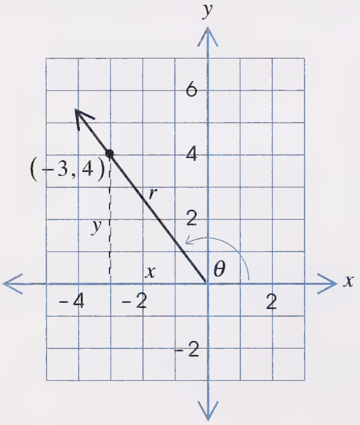
1. Draw the angle in standard position. Make a triangle by drawing a perpendicular from the point $(5, 12)$ to the x-axis.



$$r^2 = x^2 + y^2$$
$$r^2 = 5^2 + 12^2$$
$$r^2 = 25 + 144$$
$$r^2 = 169$$
$$r = \sqrt{169}$$
$$= 13$$

$$\sin\theta = \frac{y}{r} = \frac{12}{13} \qquad \cos\theta = \frac{x}{r} = \frac{5}{13}$$

2. Draw the angle in standard position. Make a triangle by drawing a perpendicular from $(-3, 4)$ to the x-axis.

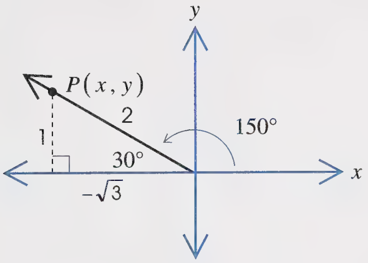


$$r^2 = x^2 + y^2$$
$$r^2 = (-3)^2 + 4^2$$
$$r^2 = 9 + 16$$
$$r^2 = 25$$
$$r = \sqrt{25}$$
$$= 5$$

$$\sin\theta = \frac{y}{r} = \frac{4}{5} \qquad \cos\theta = \frac{x}{r} = \frac{-3}{5}$$

b. Textbook questions 6 to 9 of "Practice," p. 341

6.

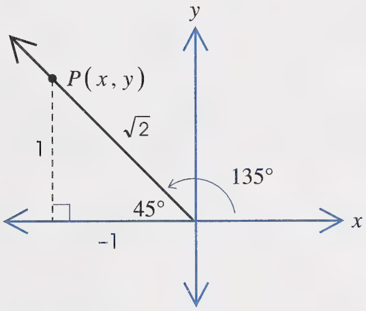


$$\sin 150° = \frac{y}{r}$$
$$= \frac{1}{2}$$

7. $\cos 150° = \frac{x}{r}$

$$= \frac{-\sqrt{3}}{2}$$

8.



$$\sin 135° = \frac{y}{r}$$
$$= \frac{1}{\sqrt{2}}$$

9. $\cos 135° = \frac{x}{r}$

$$= \frac{-1}{\sqrt{2}}$$

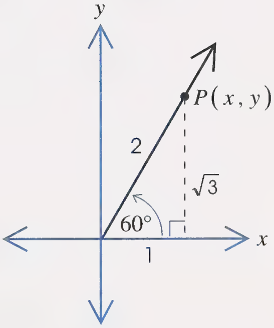
3. a. Textbook questions 4 and 5 of "Practice," p. 341

4. Since $\cos A = \frac{x}{r}$ and $\cos A = \frac{1}{2}$, then $x = 1$ if $r = 2$.

$$\therefore\ y = \sqrt{2^2 - 1^2}$$
$$= \sqrt{3}$$

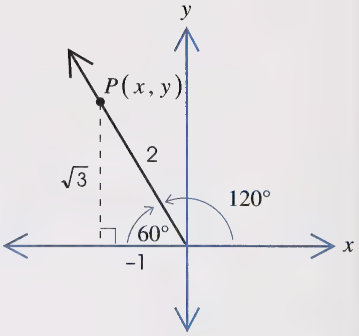
Since $r = 1$, $y = \sqrt{3}$, and $r = 2$, the triangle must be a $30°–60°–90°$ triangle.

Cosine is positive when the terminal arm is in the first quadrant and negative if it is in the second quadrant; therefore, $\angle A = 60°$.



Section 1: Activity 3 (continued)

5. Since $\cos A = \frac{x}{r}$ and $\cos A = -\frac{1}{2}$, then $x = -1$ if $r = 2$.



$$\begin{aligned} \therefore\ y &= \sqrt{2^2 - (-1)^2} \\ &= \sqrt{3} \end{aligned}$$

Since $x = -1$, $y = \sqrt{3}$, and $r = 2$, the triangle must be a $30° - 60° - 90°$ triangle.

Cosine is negative when the terminal arm is in the second quadrant.

$$\begin{aligned} \therefore\ \angle A &= 180° - 60° \\ &= 120° \end{aligned}$$

b. Textbook questions 18 to 20 of "Practice," p. 342

18. $\sin \theta = 0.2786$

$\theta \doteq 16°$

The sine ratio is positive in both the first and second quadrants. Therefore, $\angle A$ is approximately 14.9° or $180° - 14.9° \doteq 165.1°$.

19. $\cos A = 0.4561$

$\angle A \doteq 62.9°$

Cosine is positive in the first quadrant. Therefore, $\angle A$ is approximately 62.9°.

20. $\cos A = -0.5603$

$\angle A \doteq 124.1°$

Cosine is negative in the second quadrant. Therefore, $\angle A$ is approximately 124.1°.

4. Textbook questions 21 to 24 of "Applications and Problem Solving," p. 342

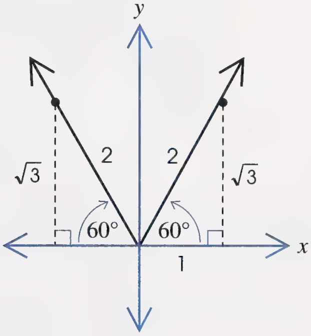
21. If the first angle is θ, the second is $\theta + 90°$ because the terminal arms are perpendicular. The angles are also supplementary. This means that the sum of these two angles is 180°.

$$\begin{aligned} \therefore\ \theta+(\theta+90°) &= 180° \\ 2\theta &= 180°-90° \\ 2\theta &= 90° \\ \theta &= 45° \end{aligned}$$

Therefore, the angles are 45° and $45°+90°=135°$.

22. If $\sin A = \sin 125°$, then $\angle A = 125°$ or $\angle A = 180° - 125° = 55°$.

23. Since $\sin\theta = \frac{y}{r}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $y=\sqrt{3}$ if $r=2$. The sine ratio is positive in both quadrants if $0 \le \theta \le 180°$.



$$\begin{aligned} \therefore\ x &= \sqrt{2^2-\left(\sqrt{3}\right)^2} \\ &= 1 \text{ or } -1 \end{aligned}$$

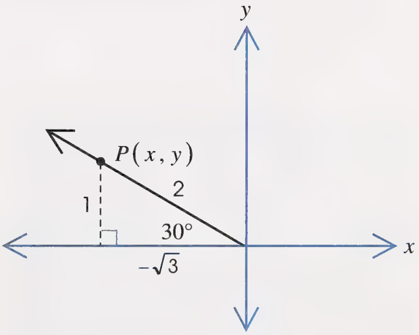
These are the sides of a $30°–60°–90°$ triangle.

a. If θ is acute, then $\cos\theta = \frac{x}{r} = \frac{1}{2}$.

b. If θ is obtuse, then $\cos\theta = \frac{x}{r} = -\frac{1}{2}$.

24. Since $\cos\theta$ is negative, the terminal arm of the angle is in the second quadrant. Since

$\cos\theta = \frac{x}{r}$ and $\cos\theta = \frac{-\sqrt{3}}{2}$, then $x = -\sqrt{3}$ if $r=2$.



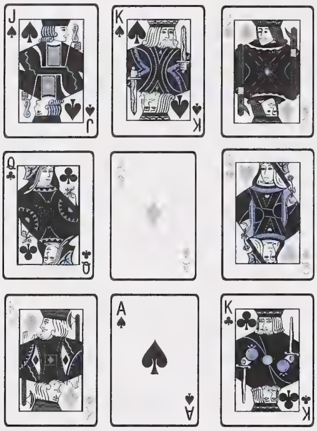
$$\begin{aligned} \therefore\ y &= \sqrt{2^2-\left(-\sqrt{3}\right)^2} \\ &= 1 \end{aligned}$$

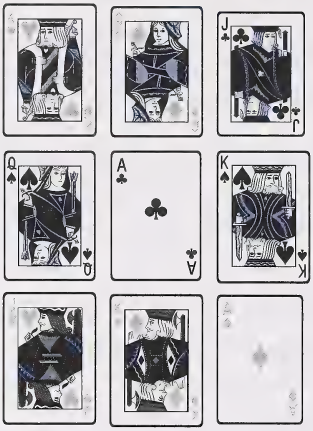
$$\begin{aligned} \therefore\ \sin\theta &= \frac{y}{r} \\ &= \frac{1}{2} \end{aligned}$$

Section 1: Activity 3 (continued)

5. **Textbook question "LOGIC POWER," p. 342**

Answers may vary. Two sample answers are given.





Section 1: Follow up Activities

Extra Help

1. Answers will vary. A sample answer is given.

$\sin 14° = \sin 166° \doteq 0.2419$

$\sin 122° = \sin 58° \doteq 0.8480$

2. $\sin\theta = \sin(180° - \theta)$

3. Answers will vary. A sample answer is given.

$\cos 74° = -\cos 106° \doteq 0.2756$

$\cos 118° = -\cos 62° \doteq -0.4695$

4. $\cos\theta = -\cos(180° - \theta)$

5. **a.** $\sin 127° \doteq 0.7986$ **b.** $\cos 11° \doteq 0.9816$ **c.** $\cos 139° \doteq -0.7547$

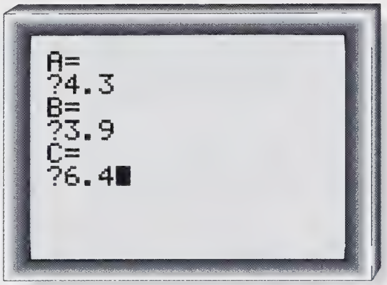
d. $\sin 68° \doteq 0.9272$ **e.** $\cos 168° \doteq -0.9781$

6. **a.** $\sin\theta = 0.2786$, $\theta \doteq 16°$ **b.** $\cos\theta = 0.3940$, $\theta \doteq 67°$ **c.** $\cos\theta = -0.5764$, $\theta \doteq 125°$ **d.** $\sin\theta = 0.9276$, $\theta \doteq 68°$

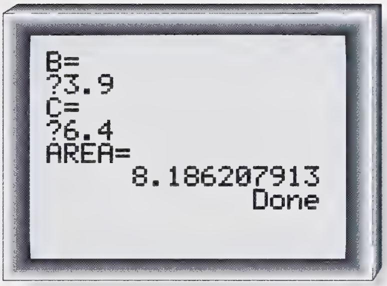
Enrichment

Textbook questions 2 and 3 of "Heron's Formula," p. 355

2. **a.**



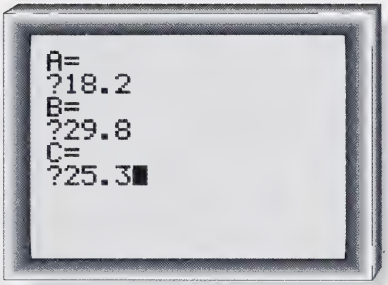
```
A=
?4.3
B=
?3.9
C=
?6.4■
```



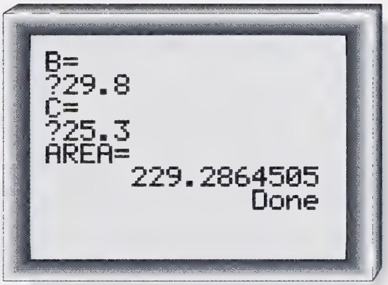
```
B=
?3.9
C=
?6.4
AREA=
         8.186207913
                Done
```

The area is approximately 8.2 m^2.

b.



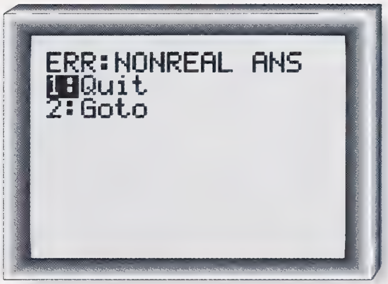
```
A=
?18.2
B=
?29.8
C=
?25.3■
```



```
B=
?29.8
C=
?25.3
AREA=
         229.2864505
                Done
```

The area is approximately 229.3 cm^2.

3. An error message appears on your calculator. It is not possible to calculate the area because the triangle does not exist. When the sum of any two sides is less than the third side, a triangle does not exist.



```
ERR:NONREAL ANS
1:Quit
2:Goto
```

Section 2: Activity 1

1. **Textbook questions 1 to 4 of "Inquire," p. 344**

   1. The altitude of the mountain is drawn at right angles to the horizontal plane. Therefore, both ΔCDA and ΔCDB are right triangles.

   2. Yes, ΔABD is oblique. The measures of its angles are 53°, 42°, and $180° - (53° + 42°) = 85°$.

   3. In ΔABD, $\angle BAD = 42°$, $\angle ABD = 53°$, and $\angle BDA = 85°$. Since AB is the side opposite the largest angle of the triangle, it is the longest side of the triangle. Therefore, AB is longer than length x and length y.

   4. **a.** In ΔCDB, $\angle CDB = 90°$.

      $$\therefore \tan 37.8° = \frac{h}{x}$$
      $$h = x \tan 37.8°$$

      b. In ΔCDA, $\angle CDA = 90°$.

      $$\therefore \tan 33° = \frac{h}{y}$$
      $$h = y \tan 33°$$

2. **Textbook questions 1, 3, 5, 7, and 9 of "Practice," p. 347**

   1. Use the law of sines to find a.

      $$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{a}{\sin 56°} &= \frac{40}{\sin 71°} \\ a &= \frac{40 \sin 56°}{\sin 71°} \\ &\doteq 35.1 \text{ cm} \end{aligned}$$

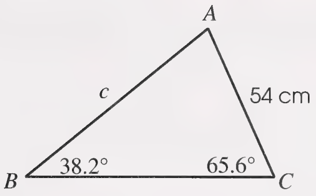
   3. First, find the measure of $\angle W$.

      $$\begin{aligned} \angle W &= 180° - (\angle X + \angle Y) \\ &= 180° - (81.3° + 67.5°) \\ &= 180° - 148.8° \\ &= 31.2° \end{aligned}$$

      Now, use the law of sines to find y.

      $$\begin{aligned} \frac{y}{\sin Y} &= \frac{w}{\sin W} \\ \frac{y}{\sin 67.5°} &= \frac{15.6}{\sin 31.2°} \\ y &= \frac{15.6 \sin 67.5°}{\sin 31.2°} \\ &\doteq 27.8 \text{ m} \end{aligned}$$

5. Use the Sine Law to find c.



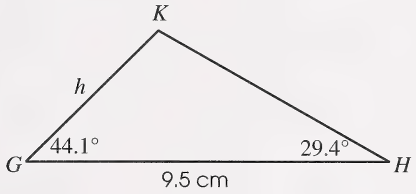
$$\frac{c}{\sin C}=\frac{b}{\sin B}$$

$$\frac{c}{\sin 65.6^\circ}=\frac{54}{\sin 38.2^\circ}$$

$$c=\frac{54\sin 65.6^\circ}{\sin 38.2^\circ}$$

$$\doteq 79.5\text{ cm}$$

7. Find the measure of $\angle K$.



$$\angle K=180^\circ-(44.1^\circ+29.4^\circ)$$

$$=180^\circ-73.5^\circ$$

$$=106.5^\circ$$

Use the law of sines to find h.

$$\frac{h}{\sin H}=\frac{k}{\sin K}$$

$$\frac{h}{\sin 29.4^\circ}=\frac{9.5}{\sin 106.5^\circ}$$

$$h=\frac{9.5\sin 29.4^\circ}{\sin 106.5^\circ}$$

$$\doteq 4.9\text{ cm}$$

9. Find the measure of $\angle R$.

$$\angle R=180^\circ-(22.2^\circ+31.4^\circ)$$

$$=180^\circ-53.6^\circ$$

$$=126.4^\circ$$

Use the Sine Law to find q.

$$\frac{q}{\sin Q}=\frac{p}{\sin P}$$

$$\frac{q}{\sin 22.2^\circ}=\frac{41}{\sin 31.4^\circ}$$

$$q=\frac{41\sin 22.2^\circ}{\sin 31.4^\circ}$$

$$\doteq 29.7\text{ cm}$$

Use the Sine Law to find r.

$$\frac{r}{\sin R}=\frac{p}{\sin P}$$

$$\frac{r}{\sin 126.4^\circ}=\frac{41}{\sin 31.4^\circ}$$

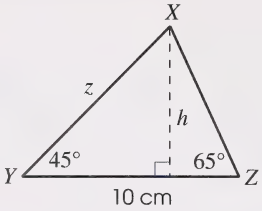
$$r=\frac{41\sin 126.4^\circ}{\sin 31.4^\circ}$$

$$\doteq 63.3\text{ cm}$$

Section 2: Activity 1 (continued)

3. a. **Textbook questions 11 and 13 of "Practice," p. 347**

11. Find the measure of $\angle X$.



$$\begin{aligned}\angle X &= 180° - (45° + 65°)\\ &= 180° - 110°\\ &= 70°\end{aligned}$$

Use the Sine Law to find z.

$$\frac{z}{\sin Z} = \frac{x}{\sin X}$$

$$\frac{z}{\sin 65°} = \frac{10}{\sin 70°}$$

$$z = \frac{10 \sin 65°}{\sin 70°}$$

Draw an altitude, h, from X, and find its length.

$$\frac{h}{z} = \sin 45°$$

$$\begin{aligned}h &= z \sin 45°\\ &= \frac{10 \sin 65°}{\sin 70°} \sin 45°\\ &= \frac{10(\sin 65°)(\sin 45°)}{\sin 70°}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Area of } \Delta XYZ &= \frac{1}{2}bh\\ &= \frac{1}{2}(10) \times \frac{10(\sin 65°)(\sin 45°)}{\sin 70°}\\ &= \frac{5(10)(\sin 65°)(\sin 45°)}{\sin 70°}\\ &\doteq 34 \text{ cm}^2\end{aligned}$$

13. Find the measure of $\angle G$.

$$\begin{aligned}\angle G &= 180° - (48.1° + 82.3°)\\ &= 180° - (130.4°)\\ &= 49.6°\end{aligned}$$

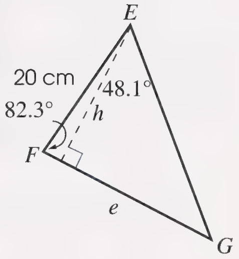
Use the Sine Law to find *e*.

$$\begin{aligned}\frac{e}{\sin E} &= \frac{g}{\sin G}\\ \frac{e}{\sin 48.1°} &= \frac{20}{\sin 49.6°}\\ e &= \frac{20 \sin 48.1°}{\sin 49.6°}\end{aligned}$$

Draw an altitude, *h*, from *E*, and find its length.

$$\begin{aligned}\frac{h}{20} &= \sin 82.3°\\ h &= 20 \sin 82.3°\end{aligned}$$

$$\begin{aligned}\therefore \text{ Area of } \Delta EFG &= \frac{1}{2}bh\\ &= \frac{1}{2}\left(\frac{20 \sin 48.1°}{\sin 49.6°}\right)(20 \sin 82.3°)\\ &\doteq 194 \text{ cm}^2\end{aligned}$$



b. Textbook questions 15, 17, 18, and 22.a. of "Applications and Problem Solving," pp. 347 and 348

15. a.
$$\begin{aligned}\angle D &= 180° - (42° + 19°)\\ &= 180° - 61°\\ &= 119°\end{aligned}$$

$$\begin{aligned}\frac{DM_1}{\sin 19°} &= \frac{1600}{\sin 119°}\\ DM_1 &= \frac{1600 \sin 19°}{\sin 119°}\\ &\doteq 596\end{aligned}$$

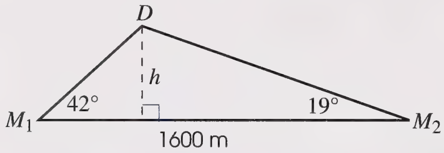
$$\begin{aligned}\frac{DM_2}{\sin 42°} &= \frac{1600}{\sin 119°}\\ DM_2 &= \frac{1600 \sin 42°}{\sin 119°}\\ &\doteq 1224\end{aligned}$$

The first museum is about 596 m from the dock. The second museum is about 1224 m from the dock.

Section 2: Activity 1 (continued)

b. To find the width of this part of the Nile, draw an altitude, *h*, from *D*.

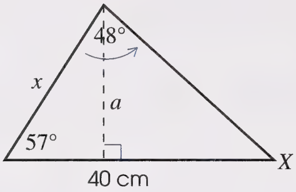
$$\frac{h}{596} \doteq \sin 42°$$
$$h \doteq 596 \sin 42°$$
$$\doteq 399$$



The width of this part of the Nile is approximately 399 m.

17. Begin by finding the area, *B*, of the triangular base of the prism.

$$\angle X = 180° - (48° + 57°)$$
$$= 180° - 105°$$
$$= 75°$$



Use the Sine Law to find *x*.

$$\frac{x}{\sin 75°} = \frac{40}{\sin 48°}$$
$$x = \frac{40 \sin 75°}{\sin 48°}$$

Draw an altitude, *a*, and find its length.

$$\frac{a}{x} = \sin 57°$$
$$a = x \sin 57°$$
$$= \frac{40 \sin 75°}{\sin 48°}(\sin 57°)$$
$$= \frac{40 \sin 75° \sin 57°}{\sin 48°}$$

$$\therefore\ B = \frac{1}{2}bh$$
$$\doteq \frac{1}{2}(40)\left(\frac{40 \sin 75° \sin 57°}{\sin 48°}\right)$$

Now, find the volume of the prism.

$$V = Bh$$
$$= \frac{1}{2}(40)\left(\frac{40\sin 75°\sin 57°}{\sin 48°}\right)(35)$$
$$\doteq 30\,522$$

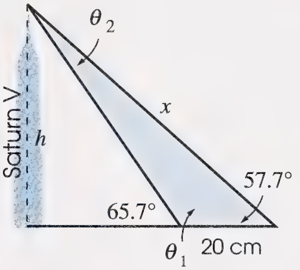
The volume of the solid is approximately 30 522 cm^3.

22. **a.** Let h be the height of the Saturn V rocket.

Begin by finding x in the shaded oblique triangle. You must find the measures of angles θ_1 and θ_2.

$$\angle\theta_1 = 180° - 65.7°$$
$$= 114.3°$$

$$\angle\theta_2 = 180° - (57.7° + 114.3°)$$
$$= 180° - 172.0°$$
$$= 8.0°$$



Use the Sine Law.

$$\frac{x}{\sin\theta_1} = \frac{20}{\sin\theta_2}$$
$$x = \frac{20\sin 114.3°}{\sin 8.0°}$$

Now find h.

$$\frac{h}{x} = \sin 57.7°$$
$$h = x\sin 57.7°$$
$$= \frac{20\sin 114.3°\sin 57.7°}{\sin 8.0°}$$
$$\doteq 111$$

The height of the Saturn V rocket is approximately 111 m.

Section 2: Activity 1 (continued)

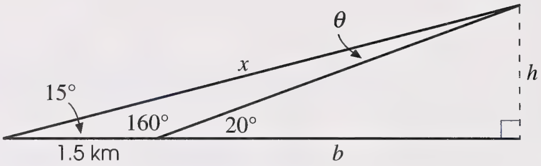
4. **a.** $\angle\theta = 180° - (15° + 160°)$

$= 5°$

$$\therefore \frac{x}{\sin 160°} = \frac{1.5}{\sin 5°}$$

$$x = \frac{1.5\sin 160°}{\sin 5°}$$

$$\doteq 5.9$$



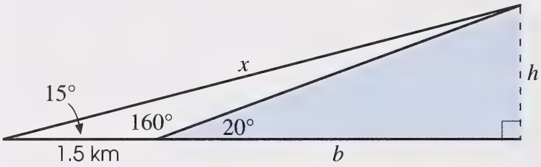
The distance from Yvanna's camp to the top of the mountain is approximately 5.9 km.

b. From the smaller shaded right triangle,

$$\frac{h}{b} = \tan 20°$$

$$h = b\tan 20°$$

$$b = \frac{h}{\tan 20°}$$



From the larger right triangle,

$$\frac{h}{1.5+b} = \tan 15°$$

$$h = (\tan 15°)(1.5+b)$$

$$h = 1.5\tan 15° + b\tan 15°$$

$$h - 1.5\tan 15° = b\tan 15°$$

$$b = \frac{h - 1.5\tan 15°}{\tan 15°}$$

Now, find *h*.

$$b = b$$

$$\frac{h}{\tan 20°} = \frac{h - 1.5\tan 15°}{\tan 15°}$$

$$h\tan 15° = (h - 1.5\tan 15°)(\tan 20°)$$

$$h\tan 15° = h\tan 20° - 1.5(\tan 15°)(\tan 20°)$$

$$1.5(\tan 15°)(\tan 20°) = h\tan 20° - h\tan 15°$$

$$1.5(\tan 15°)(\tan 20°) = h(\tan 20° - \tan 15°)$$

$$h = \frac{1.5(\tan 15°)(\tan 20°)}{\tan 20° - \tan 15°}$$

Finally, using the larger right triangle,

$$\begin{aligned}\frac{h}{x} &= \sin 15^\circ \\ h &= x \sin 15^\circ \\ x &= \frac{h}{\sin 15^\circ} \\ &= \frac{\frac{1.5(\tan 15^\circ)(\tan 20^\circ)}{\tan 20^\circ - \tan 15^\circ}}{\sin 15^\circ} \\ &= \frac{1.5(\tan 15^\circ)(\tan 20^\circ)}{\sin 15^\circ(\tan 20^\circ - \tan 15^\circ)} \\ &\doteq 5.9\end{aligned}$$

The distance from Yvanna's camp to the top of the mountain is approximately 5.9 km.

Section 2: Activity 2

1. **Textbook questions 1 to 5 of "Inquire," p. 349**

1. Because a is the hypotenuse of ΔABC, $a^2 = b^2 + c^2$.

2. **a.** If $\angle A$ is increased from a right angle to an obtuse angle, the length of side a will increase.

b. If $\angle A$ is obtuse, then $a^2 > b^2 + c^2$.

3. **a.** If $\angle A$ is decreased from a right angle to an acute angle, the length of side a would decrease.

b. If $\angle A$ is acute, then $a^2 < b^2 + c^2$.

4. **a.** If $\angle Z = 90^\circ$, then $z^2 = 5000^2 + 5500^2$

$$\begin{aligned} z &= \sqrt{5000^2 + 5500^2} \\ &\doteq 7433 \end{aligned}$$

The length of the tunnel would be approximately 7433 m.

b. If $\angle Z = 10^\circ$, side z would be less than 7433 m in length.

c. Answers may vary. A good approximation for the length of the tunnel is 800 m.

Section 2: Activity 2 (continued)

5. No, the law of sines can only be used if you are given two angles and a side or two sides and an angle opposite one of the sides. In this situation, you are given two sides and the contained angle.

2. Textbook questions 1, 3, 7, and 10 of "Practice," p. 352

1. Use the law of cosines to find *s*.

$$s^2 = r^2 + t^2 - 2rt\cos S$$
$$s^2 = 20^2 + 10^2 - 2(20)(10)\cos 40°$$
$$s^2 \doteq 193.58$$
$$s \doteq 13.9 \text{ m}$$

3. Use the law of cosines to find *m*.

$$m^2 = \ell^2 + n^2 - 2\ell n\cos M$$
$$m^2 = 133^2 + 97^2 - 2(133)(97)\cos 121°$$
$$m^2 \doteq 40\,387.01$$
$$m \doteq 201.0 \text{ m}$$

7. Use the law of cosines to find *r*.

$$r^2 = p^2 + q^2 - 2pq\cos R$$
$$r^2 = 15.3^2 + 18.2^2 - 2(15.3)(18.2)\cos 70°$$
$$r^2 \doteq 374.85$$
$$r \doteq 19.4 \text{ m}$$

10. Use the law of cosines to find *n*.

$$n^2 = m^2 + o^2 - 2mo\cos N$$
$$n^2 = 36.5^2 + 51.4^2 - 2(36.5)(51.4)\cos 126°$$
$$n^2 \doteq 6179.70$$
$$n \doteq 78.6 \text{ m}$$

3. Textbook questions 5, 6, 9, and 11 of "Practice," p. 352

5. Use the law of cosines to find $\angle C$.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{16^2 + 17^2 - 14^2}{2(16)(17)}$$

$$\angle C \doteq 50.1°$$

6. Use the Cosine Law to find $\angle E$.

$$\cos E = \frac{d^2 + f^2 - e^2}{2df}$$

$$\cos E = \frac{14.7^2 + 15.3^2 - 28.1^2}{2(14.7)(15.3)}$$

$$\angle E \doteq 139.0°$$

9. Use the Cosine Law to find $\angle B$.

$$\cos B = \frac{c^2 + d^2 - b^2}{2cd}$$

$$\cos B = \frac{22.1^2 + 22.1^2 - 35.6^2}{2(22.1)(22.1)}$$

$$\angle B \doteq 107.3°$$

11. Use the Cosine Law to find $\angle K$.

$$\cos K = \frac{\ell^2 + m^2 - k^2}{2\ell m}$$

$$\cos K = \frac{2.5^2 + 3.2^2 - 2.8^2}{2(2.5)(3.2)}$$

$$\angle K \doteq 57.3°$$

Next, find $\angle L$ using the law of cosines.

$$\cos L = \frac{k^2 + m^2 - \ell^2}{2km}$$

$$\cos L = \frac{2.8^2 + 3.2^2 - 2.5^2}{2(2.8)(3.2)}$$

$$\angle L \doteq 48.7°$$

$$\begin{aligned} \therefore\ \angle M &\doteq 180° - (57.3° + 48.7°) \\ &\doteq 180° - 106.0° \\ &\doteq 74.0° \end{aligned}$$

4. Textbook questions 15, 17, 20, 23, and 28 of "Applications and Problem Solving," pp. 352 and 353

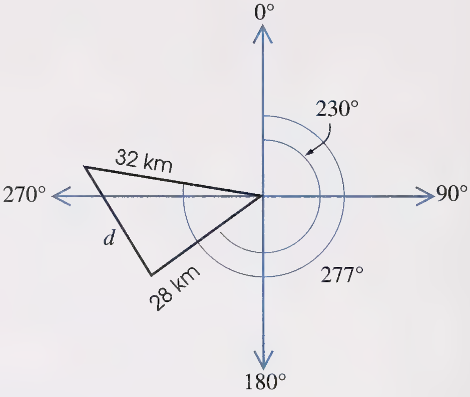
15. Let t be the distance from the ball to the hole.

$$\begin{aligned} \therefore\ t^2 &= b^2 + h^2 - 2bh\cos T \\ t^2 &= 348^2 + 195^2 - 2(348)(195)\cos 12° \\ t^2 &\doteq 26\,374.81 \\ t &\doteq 162 \end{aligned}$$

The ball is approximately 162 m from the hole.

Section 2: Activity 2 (continued)

17. The angle between the ships' courses is $277° - 230° = 47°$. After 2 h, the ships have sailed $2(16) = 32$ km and $2(14) = 28$ km, respectively.



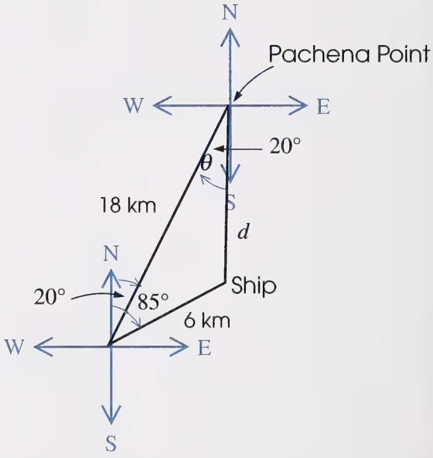
Let d represent the distance between the ships.

Using the law of cosines,

$$\begin{aligned} d^2 &= 32^2 + 28^2 - 2(32)(28)\cos 47° \\ d^2 &\doteq 585.86 \\ d &\doteq 24.2 \text{ km} \end{aligned}$$

The ships are approximately 24.2 km apart.

20. **a.** The angle between the courses marked 18 km and 6 km is $85° - 20° = 65°$.



To find the distance, d, between the ship and Pachena Point, use the law of cosines.

$$\begin{aligned} d^2 &= 6^2 + 18^2 - 2(6)(18)\cos 65° \\ d^2 &\doteq 268.71 \\ d &\doteq 16 \end{aligned}$$

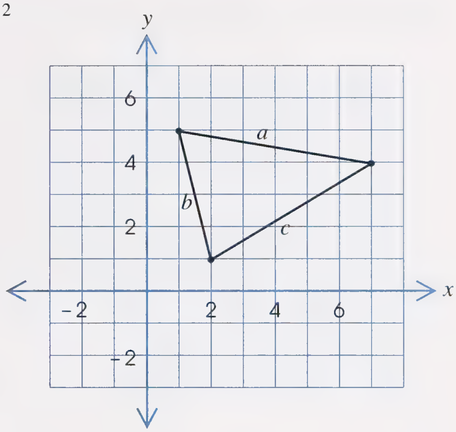
The ship is approximately 16 km from Pachena Point.

b. To determine the direction they should sail to return to Pachena Point, determine $\angle\theta$ using the Cosine Law.

$$\begin{aligned} \cos\theta &\doteq \frac{18^2 + 16^2 - 6^2}{2(18)(16)} \\ \angle\theta &\doteq 19° \end{aligned}$$

Since their original course was 20° west of south, the treasure hunters must sail about $20° - 19° = 1°$ east of north in order to return to Pachena Point.

23. Use the distance formula $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$ to determine the length of each side of ΔABC.



$$\begin{aligned} AB \text{ or } c &= \sqrt{(7-2)^2+(4-1)^2} \\ &= \sqrt{5^2+3^2} \\ &= \sqrt{34} \\ &\doteq 5.8 \end{aligned}$$

$$\begin{aligned} AC \text{ or } b &= \sqrt{(1-2)^2+(5-1)^2} \\ &= \sqrt{(-1)^2+4^2} \\ &= \sqrt{17} \\ &\doteq 4.1 \end{aligned}$$

$$\begin{aligned} BC \text{ or } a &= \sqrt{(1-7)^2+(5-4)^2} \\ &= \sqrt{(-6)^2+1^2} \\ &= \sqrt{37} \\ &\doteq 6.1 \end{aligned}$$

Use the Cosine Law to find each angle.

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\cos A=\frac{\left(\sqrt{17}\right)^2+\left(\sqrt{34}\right)^2-\left(\sqrt{37}\right)^2}{2\sqrt{17}\times\sqrt{34}}$$

$$\angle A \doteq 73.1°$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$\cos B=\frac{\left(\sqrt{37}\right)^2+\left(\sqrt{34}\right)^2-\left(\sqrt{17}\right)^2}{2\sqrt{37}\times\sqrt{34}}$$

$$\angle B \doteq 40.4°$$

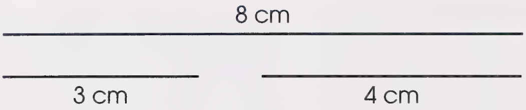
$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$\cos C=\frac{\left(\sqrt{37}\right)^2+\left(\sqrt{17}\right)^2-\left(\sqrt{34}\right)^2}{2\sqrt{37}\times\sqrt{17}}$$

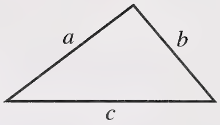
$$\angle C \doteq 66.5°$$

Section 2: Activity 2 (continued)

28.



Can you draw a triangle using these measures? The answer is no, since $8 > 3 + 4$. For any triangle, the sum of two sides must be greater than the third side. This property is called **triangle inequality**.



$a < b + c$

$b < a + c$

$c < a + b$

Look at what happens when you try to solve for an angle using 3, 4, and 8.

$$\begin{aligned}\cos\theta &= \frac{8^2 + 4^2 - 3^2}{2(8)(4)}\\ &= \frac{64 + 16 - 9}{64}\\ &= \frac{71}{64}\end{aligned}$$

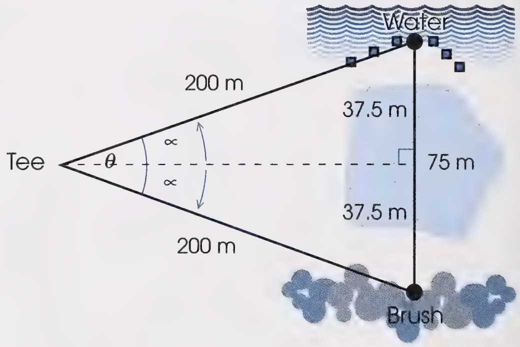
> ***Note:*** *The other two calculations give an angle measure; but when these two angles are added together, it exceeds 180°—the sum of the angles of any triangle.*

The cosines of angles between 0° and 180° lie between -1 and $+1$. Therefore, $\frac{71}{64}$ is too large. Your calculator will give you an error message when you try to find θ.

5. **Method 1: Using the Cosine Law**

$$\cos\theta = \frac{200^2 + 200^2 - 75^2}{2(200)(200)}$$

$$\angle\theta \doteq 21.6°$$



Method 2: Using Two Congruent Triangles

Divide the triangle into two congruent right triangles.

$$\sin\propto = \frac{37.5}{200}$$

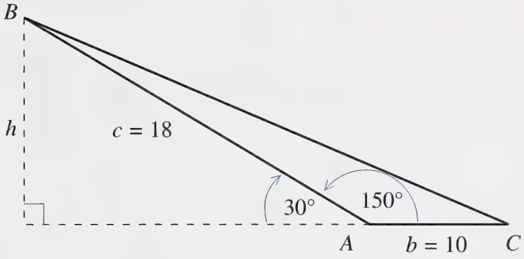
$$\angle\propto \doteq 10.8°$$

$$\begin{aligned}\therefore\ \angle\theta &= 2 \times \angle\propto\\ &\doteq 2(10.8°)\\ &\doteq 21.6°\end{aligned}$$

Section 2: Follow-up Activities

Extra Help

1. Find the height, h, of ΔABC.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30° = \frac{h}{18}$$

$$h = 18\sin 30°$$

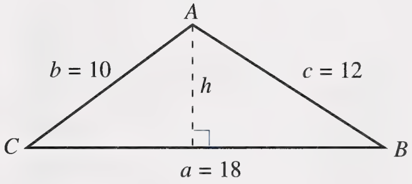
$$= 9 \text{ units}$$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}bh$$

$$= \frac{1}{2}(10)(9)$$

$$= 45 \text{ square units}$$

2. Find $\angle C$ using the law of cosines.



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{18^2 + 10^2 - 12^2}{2(18)(10)}$$

$$C \doteq 38.94°$$

Draw an altitude, h, from A.

$$\frac{h}{b} = \sin C$$

$$h = b\sin C$$

$$= 10\sin 38.94°$$

$$\doteq 6.29 \text{ units}$$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\times\text{base}\times\text{height}$$

$$\doteq \frac{1}{2}(18)(6.29)$$

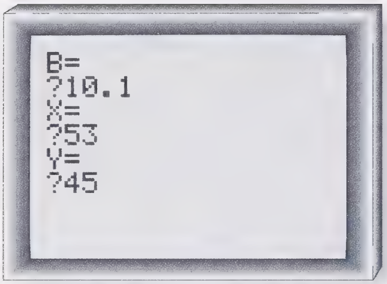
$$\doteq 56.6 \text{ square units}$$

Enrichment

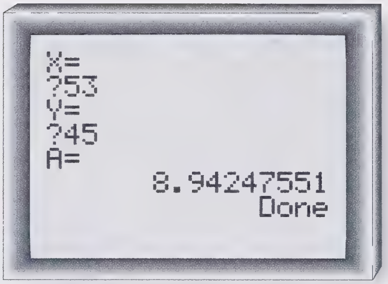
1. **Textbook question 2 of Investigation 1, “The Law of Sines,” p. 354**

   2. The answers given here show two calculator displays. This is to show you all of the information entered and the results obtained.

      a.



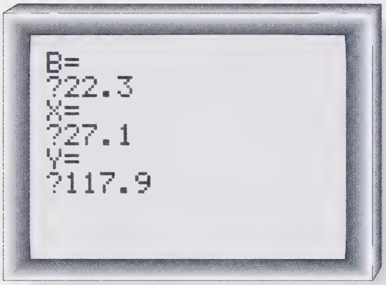
```
B=
?10.1
X=
?53
Y=
?45
```



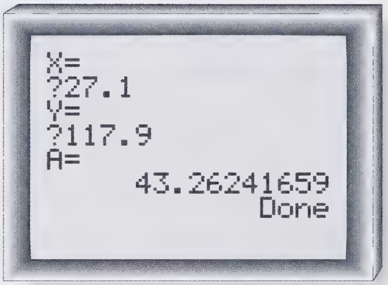
```
X=
?53
Y=
?45
A=
          8.94247551
                Done
```

$\therefore A \doteq 8.9$ cm

b.



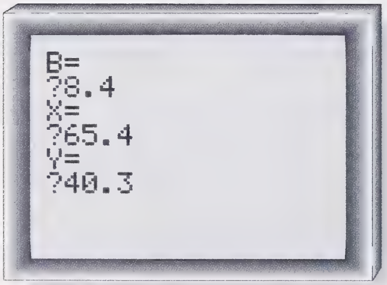
```
B=
?22.3
X=
?27.1
Y=
?117.9
```



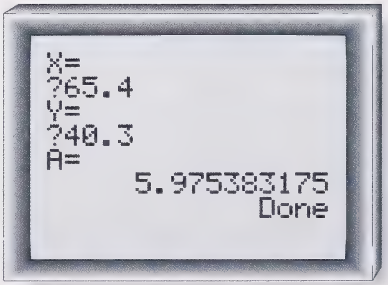
```
X=
?27.1
Y=
?117.9
A=
         43.26241659
                Done
```

$\therefore A \doteq 43.3$ m

c.



```
B=
?8.4
X=
?65.4
Y=
?40.3
```



```
X=
?65.4
Y=
?40.3
A=
         5.975383175
                Done
```

$\therefore A \doteq 6.0$ cm

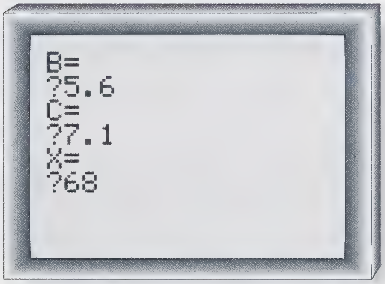
2. Textbook questions 1 to 4 of Investigation 2, “The Law of Cosines,” pp. 354 and 355

1.

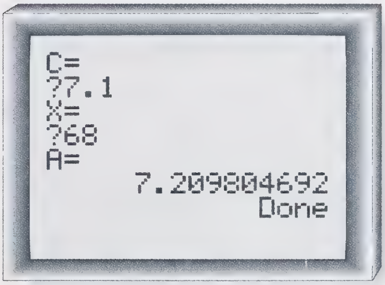
Program Line	What Each Line Does
PROGRAM : COSINES	names the program
: ClrHome	clears the screen
: Disp “B =”	prompts you to enter the length of one of the two sides given
: Input B	stores the side length you entered as variable B
: Disp “C =”	prompts you to enter the length of the other given side
: Input C	stores the side length you entered as variable C
: Disp “X =”	prompts you to enter the measure of the included angle
: Input X	stores the angle measure as variable X
: $\sqrt{}\left(B^2+C^2-2BC\cos(X)\right)\to A$	calculates the length of the third side using the cosine rule and stores the value as variable A
: Disp “A =”	prints A = on the screen
: Disp A	prints the value of A on the screen

2. The answers given here show two calculator displays. This is to show you all of the information entered and the results obtained.

a.



```
B=
?5.6
C=
?7.1
X=
?68
```

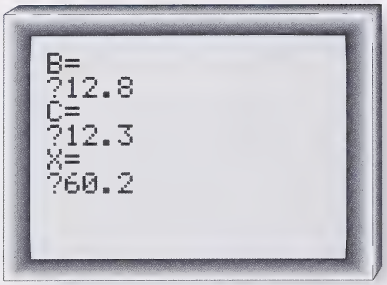


```
C=
?7.1
X=
?68
A=
            7.209804692
                   Done
```

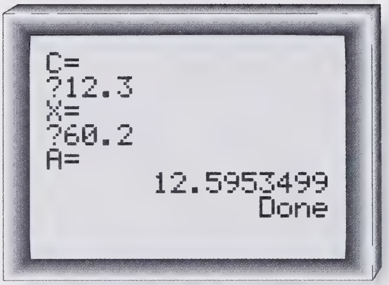
$\therefore\ A \doteq 7.2\ \text{cm}$

Enrichment (continued)

b.



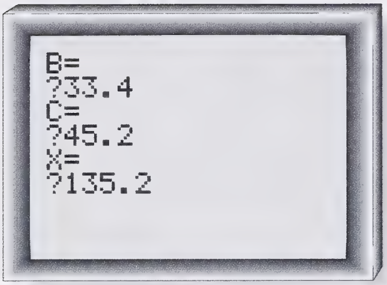
```
B=
?12.8
C=
?12.3
X=
?60.2
```



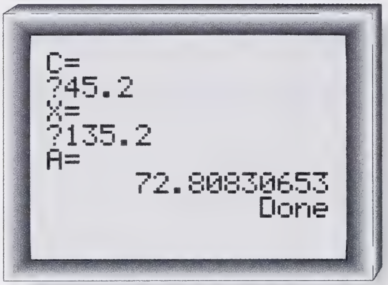
```
C=
?12.3
X=
?60.2
A=
          12.5953499
                Done
```

$\therefore\ A \doteq 12.6\ \text{m}$

c.



```
B=
?33.4
C=
?45.2
X=
?135.2
```



```
C=
?45.2
X=
?135.2
A=
         72.80830653
                Done
```

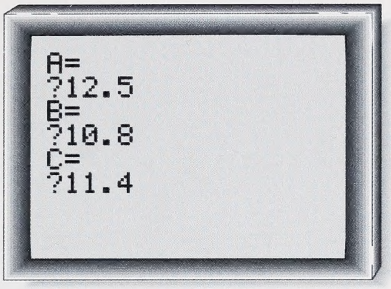
$\therefore\ A \doteq 72.8\ \text{m}$

3.

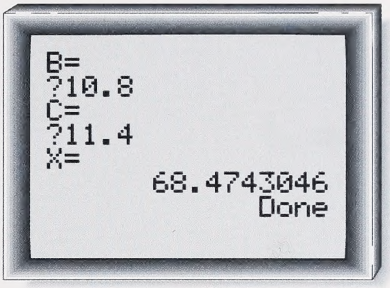
```
PROGRAM : COSINES3
: ClrHome
: Disp "A ="
: Input A
: Disp "B ="
: Input B
: Disp "C ="
: Input C
: cos⁻¹((B²+C²−A²)/(2BC))→X
: Disp "X ="
: Disp X
```

4. The answers given here show two calculator displays. This is to show you all of the information entered and the results obtained.

a.



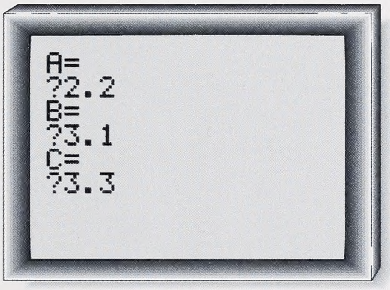
```
A=
?12.5
B=
?10.8
C=
?11.4
```



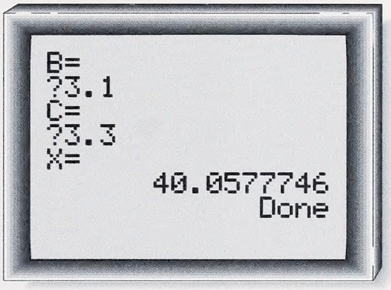
```
B=
?10.8
C=
?11.4
X=
          68.4743046
                Done
```

$\therefore X \doteq 68.5°$

b.



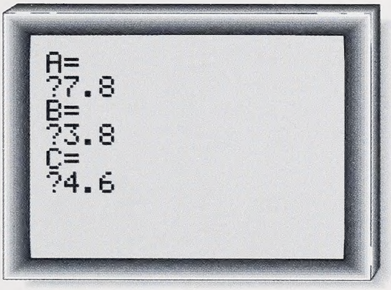
```
A=
?2.2
B=
?3.1
C=
?3.3
```



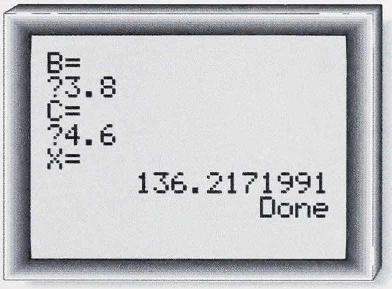
```
B=
?3.1
C=
?3.3
X=
          40.0577746
                Done
```

$\therefore X \doteq 40.1°$

c.



```
A=
?7.8
B=
?3.8
C=
?4.6
```



```
B=
?3.8
C=
?4.6
X=
         136.2171991
                Done
```

$\therefore X \doteq 136.2°$

© 2002